Psychological Review
1997, Vol. 104, No. 3, 595–602

0033-295X/97/$3.00

# Serial Exhaustive Models Can Violate the Race Model Inequality: Implications for Architecture and Capacity

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Classical examples of simple architectures, ranked in terms of their relative efficiency or capacity from less to more, are standard serial (1-at-a-time) processes, followed by independent parallel (simultaneous) processes, followed by coactive (pooling of parallel channel information), and interactive parallel (e.g., mutually facilitory) processes. Violation of the *race model inequality* has been thought to rule out ordinary parallel and serial systems. Violation of the race model inequality does exclude many such processes. However, it is proven here that a variety of serial systems that is quite inefficient readily violates the race model inequality. In the Discussion, the authors indicate that the race model inequality still can be highly useful in the identification of mental architecture, when allied with other converging analyses of structure, process rules, and capacity.

The identification of mental architecture in perception and cognition remains of high import. Chief among the simple architectures are parallel and serial processing, in which processing is either simultaneous or one-at-a-time (Egeth, 1966; Sternberg, 1966). Early quantitative work, although pointing out certain directions for parallel versus serial experimental discrimination, emphasized problems of model mimicking between these two diametrically opposed types of processing as well as simply understanding the underlying stochastic processes (Atkinson, Holmgren, & Juola, 1969; Christie & Luce, 1956; McGill, 1963; Townsend, 1969, 1972).

Due to progress in the quantitative study of serial and parallel processing and in implied methodologies, these are now open to rigorous empirical testing (Ashby, Tein, & Balakrishnan, 1993; Egeth & Dagenbach, 1991; Egeth, Folk, & Mullin, 1988; Townsend, 1976a, 1976b, 1984, 1990) within certain broadly defined conditions (Ashby & Townsend, 1980; Dzhafarov & Schweickert, 1995; Luce, 1986; Ratcliff, 1978; Roberts & Sternberg, 1992).[1] Response times (RTs) and accuracy have been the primary dependent variables under study, and the former is our focus in this article.

This study points out a heretofore unknown impediment concerning one of the most promising current parallel–serial discriminative tools and how it relates to contemporary stochastic theories of information processing. Then potential devices to surmount this difficulty are discussed.

Another critical dimension, apart from architecture, in our processing taxonomy refers to the *stopping rule,* that is, the rule or condition that causes processing to cease. For instance, when a target that determines a response is found, can the processor stop, or must it continue through all the items? The former is referred to as *self-terminating processing,* whereas the latter is called *exhaustive processing.* Mathematical work has also allowed definite conclusions to be drawn from brief visual display and short-term memory search, regarding whether processing is exhaustive or self-terminating (Townsend & Colonius, in press; Townsend & Van Zandt, 1990; Van Zandt & Townsend, 1993). A special case of self-termination is when every item is a target and the very first item to be completed stops the processing. In the case of simultaneous processing, the latter defines the minimum processing time and is termed a parallel "race."

In addition to the issues of the form of the architecture and type of stopping rule, another important dimension of processing concerns how RTs are affected as the processing load is altered, for instance, by varying the number of displayed items that must be searched. *Capacity* is the term for a theoretical concept that relates variations of load to efficiency of processing. Capacity can be evaluated at a number of distinct levels (Townsend, 1974; Townsend & Ashby, 1978). A coarse and highly observable level regards how RT changes in, say, a rapid search of short-term memory or a visual display, as the total number of items input to the system (the load) increases. In general, we expect serial processing to be of limited capacity in RT at this level, relative to parallel processing, although there are cases where either may mimic the other (Townsend, 1972, 1974; Townsend & Ashby, 1983).

It has proven helpful to define a model or system's primary capacity at a finer-grained level, namely, the level of speed (or

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This research was in part supported by National Science Foundation Grant BNS 9112813.

Helpful commentaries were provided by Kam Silva and Michael Wenger.

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[1] In fact, in recent years, what we might call mathematical metatheories have lead to the development of allied experimental methodologies that can, in principle, identify and separate large classes of architectures. It is becoming feasible to identify architectures of rather exceptional complexity (Fisher & Goldstein, 1983; Goldstein & Fisher, 1991; Luce, 1986; Schweickert, 1978; Schweickert & Townsend, 1989; Schweickert & Wang, 1993; Townsend & Colonius, in press; Townsend & Schweickert, 1985).

processing time) of an individual item (Townsend, 1974; Townsend & Ashby, 1983, pp. 76–98). Limited capacity is defined by an increase in item-processing time as the load increases. Unlimited capacity is defined as unchanged processing rates at this individual item level as the load increases. For instance, the *fixed-capacity parallel model*, which is a special case of limited capacity, assumes that the available capacity is spread out over all the items to be processed but not necessarily in an even manner (Townsend & Ashby, 1983, p. 87). In the event that the spread is even and the overall capacity is $C$, each of $n$ input items would receive $C/n$ of the capacity, and so on.

Super capacity is defined as an actual increase in processing speed on individual items, as load is incremented. Although a theoretical necessity (e.g., Townsend, 1974; Townsend & Ashby, 1983), it has heretofore been treated as of little practical interest, because few if any tasks are known where humans evidence improved RTs with increased workload. However, we shall find the concept beneficial in what follows (see also, Townsend & Nozawa, 1995). Observe that at the individual item level, standard serial processing (i.e., with equal processing rates on all items and orders of processing, and different numbers of items to process) is unlimited capacity, because the processing rates at that level are invariant across load. However, it is very limited at the overall RT level, in the case of self-terminating (with one target among $n - 1$ other items) or exhaustive processing, because RT increases dramatically (in fact, linearly with nonzero slope) as the load grows in either case.

A variety of experimental paradigm that has served to falsify serial and support parallel processing is the *redundant targets design* (Egeth, Folk, & Mullin, 1988). A popular version of the redundant targets design possesses trials of three types: (a) either of two targets, but not both appear; (b) both targets appear; or (c) neither target appears. In general, the nontarget positions may be occupied by an actual nontarget pattern or no stimulus at all. The observers are instructed to give a "yes" (or some designated) response if the trial type is either a or b and "no" only if neither target appears (i.e., Type c). Clearly, if the participant can self-terminate processing when the first of the two targets is completed (Type b), then the statistic reflecting processing time is the minimum of completing either target. When processing is taking place in parallel on two channels then on redundant target trials b, this minimum processing time can be thought of as designating the winner of a parallel race.

It may be noted that the redundant targets design is a variation on the theme of setting up experimental conditions to permit different stopping rules to potentially be applied by observers without loss of accuracy. Minimum time processing is the fastest of the various stopping rules (Townsend & Ashby, 1983). If capacity is not degraded as the number of targets grows, the mean minimum processing time actually evidences a decrease as this occurs (Townsend & Ashby, 1983, Figure 4.5).

The canonical race model involves stochastically independent (defined by the processing times in the two channels being stochastically independent), unlimited capacity (again, the processing speed on each channel is unaffected by the presence or absence of processing on the other channel), parallel processing (Dzhafarov, 1993; Grice, Canham, & Boroughs, 1984; Marley & Colonius, 1992; Townsend, 1974; Townsend & Ashby, 1983, pp. 272–280).

This canonical parallel race model makes the prediction for redundant target trials that the minimum time cumulative distribution function (the probability that the minimum processing time is less than or equal to any given time) is equal to the sum of the separate cumulative distribution functions minus their product. This prediction is known as *probability summation*. The additional stipulation of unlimited capacity implies that each of the separate channel (or item) probabilities is the same as if it were working alone (i.e., had an item to process).

Probability summation yields performance superior to either target alone, a finding of *redundancy gain*. This redundancy gain is produced by purely statistical effects, because the individual processing rates, by assumption, are unchanged. A useful analogy is to inspect separate series of tossing one, two, three, or more coins simultaneously and, in each series, agree to stop as soon as the first head is observed among any of the simultaneously tossed coins. Each coin represents a given channel or item. On the average, the more coins are tossed, the earlier the first head appears (analogous to a detect response on a given channel), even though the probability of a head on any individual coin is unaltered (the analogy to unlimited capacity; see also Townsend, 1974, for extensions of this analogy).

It seems reasonable to allow more general race models to possess stochastically dependent channels where the two channels may interact within a processing trial. However, it is also reasonable to put some constraints on the effects of such interdependencies. The needed constraint in the present circumstances is *context independence* (Ashby & Townsend, 1986; Colonius, 1990b; Luce, 1986). Context independence is, in turn, defined by the requirement that the marginal probability distribution on completion times on each channel be unaffected by the processing occurring or not occurring on the other channel. For instance, a special case of context independence is the unlimited capacity associated with an increasing processing load. In other words, incrementing the number of channels in operation (number of presented items, etc.) does not affect the (marginal) distribution on processing times of the specified channel. We therefore define the general class of context independent race models, which assumes parallelism but allows stochastic dependence.

Miller (1978, 1982, 1986, 1991) proposed a valuable test of all parallel race models referred to as the *race model inequality*.[2] Basically, the inequality is used to see if processing on redundant target trials is very fast relative to what could be expected from the signal target trials and even in comparison with probability summation.

In order to establish notation for our development, let Condition AB refer to trials on which both channels contain a target, Condition A$\bar{\text{B}}$ to trials on which the target appears in Channel A but a nontarget is in Channel B, Condition $\bar{\text{A}}$B to trials on which the target is in Channel B but not A, and finally, Condition $\bar{\text{A}}\bar{\text{B}}$ to trials on which the target is in neither channel. In general, a nontarget could either be a blank (no specific stimulus presented in that locus by the experimenter) or a stimulus pattern that is not defined as a target.

Next, let $P_{\text{AB}}$ denote the probability measures applying in Condition AB and similarly for the other conditions. Set $T$ as the overall decision time, whereas $T_i$ (i = A, B, $\bar{\text{A}}$, $\bar{\text{B}}$) is the

[2] This inequality is known in mathematics as *Boole's inequality*.

time to process a target or nontarget, respectively, in Channel i. Thus, $P_{AB}(T \leq t)$ is the cumulative probability that the overall time on a redundant target trial is less than or equal to $t$, whereas $P_{AB}(T_A \leq t)$ is that probability specifically for the processing time in Channel A, when both channels are occupied by a target. Similar reasoning provides that $P_{A\bar{B}}(T \leq t)$ is the cumulative probability for the overall time when only Channel A possesses a target and $P_{A\bar{B}}(T_A \leq t)$ is the cumulative probability on the actual processing time in Channel A in that condition, and so on.

Observe the impact of context independence because it forces $P_{A\bar{B}}(T_A \leq t) = P_{AB}(T_A \leq t)$ always (i.e., for all times $t$), and likewise $P_{\bar{A}B}(T_B \leq t) = P_{AB}(T_B \leq t)$. That is, the probability that a target has completed processing by any time $t$ is the same whether a target or nontarget occupies the other channel.

The expression for a general race model that does not assume stochastic independence for a redundant target trial is $P_{AB}(T \leq t) = P[\mathrm{MIN}(T_A, T_B) \leq t] = P_{AB}(T_A \leq t) + P_{AB}(T_B \leq t) - P_{AB}(T_A \leq t \text{ and } T_B \leq t)$. This equation obviously implies that $P[\mathrm{MIN}(T_A, T_B) \leq t] \leq P_{AB}(T_A \leq t) + P_{AB}(T_B \leq t)$. But, by the assumption of context independence, $P_{AB}(T_A \leq t) = P_{A\bar{B}}(T_A \leq t)$ and $P_{AB}(T_B \leq t) = P_{\bar{A}B}(T_B \leq t)$. Hence, $P_{AB}[\mathrm{MIN}(T_A, T_B) \leq t] \leq P_{A\bar{B}}(T_A \leq t) + P_{\bar{A}B}(T_B \leq t)$. Now according to the race model, we have the following expressions for $T$:

$$\text{Condition AB: } T = \mathrm{MIN}\ (T_A, T_B)$$

$$\text{Condition A}\bar{\text{B}}\text{: } T = T_A$$

$$\text{Condition }\bar{\text{A}}\text{B: } T = T_B.$$

Therefore,

$$P_{AB}(T \leq t) \leq P_{A\bar{B}}(T \leq t) + P_{\bar{A}B}(T \leq t). \qquad (1)$$

This equation, Equation 1, is the race model inequality.

It is clear that the inequality must hold for sufficiently long processing times because the left-hand side of Equation 1 is bounded by 1, whereas the far right-hand side converges toward 2. Hence, the investigation looks for violations in faster RTs.

Observe that when the nontarget is just a blank stimulus, then context independence implies unlimited capacity because it assumes that an increased processing load (i.e., two rather than one target, especially pertinent if the nontargets are just blanks) does not slow processing down, as several authors have pointed out (Ashby & Townsend, 1986, p. 171; Colonius, 1990b, p. 266; Luce, 1986, p. 130).[3] Furthermore, the inequality is conservative in the sense that it is not usually expected that capacity improves with the addition of more things to process such that, if anything, one might expect the inequality to hold on those grounds alone, even without dropping the negative term. That is, violation of context independence would be expected to be in the direction of limited capacity, if there is a departure from context independence, and limited capacity would tend in the direction of satisfaction of the inequality (see Footnote 3).

In fact, ordinary parallel racing cannot produce a violation of this inequality without assuming super capacity (Townsend & Nozawa, 1995), as is further discussed later. An observed violation may implicate some type of *coactivation*, as suggested by Miller (1982, 1991). The idea of coactivation is suggestive of a pooling or convergence of information or activation into a common subsequent channel and is examined further later.

The advent of the race model inequality was of real import because, for the first time, it provided a benchmark of processing efficiency other than probability summation or serial versus unlimited capacity parallel activity. In fact, this benchmark appears to suggest that ordinary parallel processing (resulting in probability summation), much less a serial mechanism, does not suffice when the inequality is violated.

As remarked earlier, Townsend and Nozawa (1988, 1995; also see Nozawa, 1989, 1992; Nozawa, Reuter-Lorenz, & Hughes, 1994) have developed a mathematical theory that interprets parallel and coactive processing in terms of the capacity concept. Defining an index of capacity, they have shown that the race inequality can be violated if and only if their capacity index exhibits super capacity over a significant range of processing times. They also show that one well-defined form of coactivation, which produces both super capacity and violations of the race inequality, is a set of models assuming the convergence (i.e., pooling) of information that enters a "summer" or counter from the separate parallel channels. Tests of natural special cases of this class of models have been provided by Colonius (1990a), Diederich and Colonius (1991), and Schwarz (1989). Thus, compatible with the intuitive notion of coactivation, the target detector's processing time is determined by a final common channel rather than the minimum time of two separate channels.

Now, Colonius (1990b) has demonstrated that all context independent race models (i.e., parallel architecture without coactivation) obey the race model inequality. Hence, any race models that violate Equation 1 must violate context independence in a way that yields super capacity according to the Townsend and Nozawa (1995) theory. In fact, Colonius and Townsend (1992) have shown that context-dependent race models can indeed mimic coactive models, as was suggested by simulations of a special type of interactive (and therefore channel-dependent) parallel model by Mordkoff and Yantis (1991). That is, parallel race models that permit stochastic dependence, for example, by using interchannel cross-talk, can (but do not have to) violate context independence in a way that violates the race model inequality.

Thus, violation of the race model inequality logically implies super capacity when interpreted in terms of the capacity index. And in fact, from the inception of this inequality, the primary purpose of its violation has been taken as a demonstration of a type of processing more efficient even than unlimited-capacity parallel race processing.

It therefore came as a surprise, when we recently discovered a form of architecture and process stopping rule that is antipodal to super capacity parallel, or coactive processing, that permits violation of the race model inequality. The architecture is serial, that is, one-at-a-time, and the stopping rule is exhaustive, that is, both channels are always checked or processed, whether or not one or more targets are present. It is important to recall in this connection that serial exhaustive processing generally produces an appearance of very limited capacity at the overall RT level. And, in fact, parallel models that mimic the predictions

[3] Actually, the inequality is also satisfied even if the marginal distribution functions, when both targets are present, are less than when either signal was presented alone, a form of limited capacity.

of serial models in experiments requiring, for instance, exhaustive processing, would exhibit limited capacity (Townsend, 1972; Townsend & Ashby, 1983, p. 81; Townsend & Van Zandt, 1990).

The only deviation from standard serial processing in the model that violates the race model inequality is that it must be assumed that checking the nontarget channels takes more time (stochastically speaking) than do the target channels. However, this assumption is far from unusual, either in experimental findings or in theoretical assumptions (e.g., Bamber, 1969; Krueger, 1978; Van Zandt & Townsend, 1993).

Our theorem is, of course, a result in model mimicking. Frequent interesting suggestions concerning production of results, for instance, redundancy gains, by opposing models appear in the literature and must be carefully considered. However, they are often verbal in nature and hence difficult to evaluate. Simulations of a model's hypothetical actions are the next step, and a large one, in rigor. A striking example of the use of simulation in modeling mimicking is Mordkoff's (1992) demonstration of the nonutility of Grice et al.'s (1984) combination-rule regression analysis to discriminate among models capable of explaining the redundant gains effects. More robust yet, are numerical calculations based on exact formulas derived within a quantitative model. Most rigorous are arguments using closed-form proofs, where they are possible. The latter approach is taken in this note.

The rather paradoxical result that we have discovered is captured in the following proposition[4]:

> *Proposition:* Assume exhaustive, stochastically independent, serial processing and that context independence is in force. Then there exist distributions with stochastically faster times for targets than nontargets, where violations of the race model inequality are found.

*Proof:* First, we require a modified version of the race model inequality, one interpreted for serial exhaustive processes. For a serial exhaustive model, we have

$$\text{Condition AB: } T = T_A + T_B$$

$$\text{Condition A}\bar{\text{B}}\text{: } T = T_A + T_{\bar{B}}$$

$$\text{Condition }\bar{\text{A}}\text{B: } T = T_{\bar{A}} + T_B.$$

Hence, the race model inequality (Equation 1) may be reexpressed

$$P_{AB}(T_A + T_B \le t) \le P_{A\bar{B}}(T_A + T_{\bar{B}} \le t) + P_{\bar{A}B}(T_{\bar{A}} + T_B \le t). \quad (2)$$

Next, define the marginal cumulative distribution functions

$$\left.\begin{aligned} F_A(t) &= P_{AB}(T_A \le t) \\ F_B(t) &= P_{AB}(T_B \le t) \end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned} F_{\bar{A}}(t) &= P_{\bar{A}B}(T_{\bar{A}} \le t) \\ F_{\bar{B}}(t) &= P_{A\bar{B}}(T_{\bar{B}} \le t). \end{aligned}\right\} \quad (4)$$

Let $f_A(t)$, $f_B(t)$, $f_{\bar{A}}(t)$, and $f_{\bar{B}}(t)$ denote the corresponding probability density functions. Note that these assignments make sense because context independence is assumed. From here on, stochastic independence of channels is assumed. Then, the left-hand side of Equation 2 becomes

$$\begin{aligned} &P_{AB}(T_A + T_B \le t) \\ &= \int_0^t \frac{d}{dt'}[P_{AB}(T_A \le t')]P_{AB}(T_B \le t - t')dt' \\ &= \int_0^t f_A(t')F_B(t - t')dt'. \end{aligned} \quad (5)$$

The last expression uses the first part of Equation 3. Next, note that

$$P_{A\bar{B}}(T_A + T_{\bar{B}} \le t) = \int_0^t \frac{d}{dt'}[P_{A\bar{B}}(T_A \le t')]P_{A\bar{B}}(T_{\bar{B}} \le t - t')dt'. \quad (6)$$

Again assuming context independence and then applying the second part of Equation 4,

$$P_{A\bar{B}}(T_A \le t') = P_{AB}(T_A \le t') = F_A(t'),$$

and so,

$$\frac{d}{dt'}P_{A\bar{B}}(T_A \le t') = f_A(t'). \quad (7)$$

Next, replacing the terms on the right hand side of Equation 6 with those given by Equations 4 and 7,

$$P_{A\bar{B}}(T_A + T_{\bar{B}} \le t) = \int_0^t f_A(t')F_{\bar{B}}(t - t')dt'. \quad (8)$$

In an analogous manner, assuming context independence, it can be shown that

$$P_{AB}(T_{\bar{A}} + T_B \le t) = \int_0^t f_B(t')F_{\bar{A}}(t - t')dt'. \quad (9)$$

Finally, substituting Equation 5 on the left-hand side of Equation 2 and Equations 8 and 9 in the right-hand side of Equation 2, we produce the representation,

$$\begin{aligned} P(T_A + T_B \le t) &= \int_0^t f_A(t')F_B(t - t')dt' \\ &\le \int_0^t f_A(t')F_{\bar{B}}(t - t')dt' + \int_0^t f_B(t')F_{\bar{A}}(t - t')dt'. \end{aligned} \quad (10)$$

This interpretation of the race model inequality for serial exhaustive models can be rewritten as:

$$\int_0^t f_A(t')[F_B(t - t') - F_{\bar{B}}(t - t')]dt' \le \int_0^t f_B(t')F_{\bar{A}}(t - t')dt'.$$

It now becomes evident that if, say, we have $F_A$, $F_B$ of the same

[4] The assumption of stochastic independence of the serial processing times found in the theorem is convenient and common but can easily be dispensed with using a little additional complexity of notation.

order of magnitude and both are very much greater than $F_{\bar{A}}$, and $F_{\bar{B}}$, for an interval of time, then the left-hand side could easily be bigger than the right-hand side and thus violate the above inequality. This becomes even more obvious when we let (although we need not) $F_A$ be equivalent to $F_B$ and $F_{\bar{A}}$ to $F_{\bar{B}}$, resulting in the simplified race inequality, $\int_0^t f_A(t')[F_A(t - t') - 2F_{\bar{A}}(t - t')]dt' \leq 0$, which again, is readily seen to be violatable if $F_A(t)$ is much larger than $F_{\bar{A}}(t)$ over an interval, say, $(0, t)$. ■

It may be observed, just as with parallel models, that in the absence of context independence the race model inequality need not hold. That is, there are then no a priori constraints on the relationship of the probability distributions on the two sides of the proposed inequality. The fascinating thing is that although context independence ensures that it does hold for parallel race models, the proposition shows that context independence does not provide such assurance in the serial exhaustive case!

A simple example is found in an exhaustive serial exponential model with $f_A(t) = 100e^{-100t} = f_B(t)$ and $f_{\bar{A}}(t) = e^{-t} = f_{\bar{B}}(t)$. Thus, an easy computation shows that at $t = .06$ units, the race model inequality is violated by a striking .89. Furthermore, the range of violation extends from at least $t = 0.01$ to $t = 0.70$. Of course, the target processing rate is 100 times as fast as the nontarget rate. The important point is that violation of the inequality does not categorically exclude exhaustive serial processing, an extremely slow form of processing. And, it is to be expected that smaller rates and differences also cause violations, albeit not such sizeable ones. It should also be noticed that the present serial models do not require any type of synergistic interaction among items (really a type of underlying parallel influence) as in a serial processor outlined by Fournier and Eriksen (1990). They are true serial mechanisms, only with some heterogeneity in processing rates.

## Discussion

### *Resolution of the Apparent Paradox*

The key to the resolution of the apparent paradox is to observe that the bedrock of the race model inequality (Equations 1 and 10) lies in the comparison of the single target trials to double (i.e., redundant) target trials. Thus, even in the exhaustive serial case, the processing on the redundant targets trials in actuality demonstrates super capacity relative to the single target data. The upshot here is that race inequality effects must be interpreted in terms of relative capacity effects in comparing single to multiple target conditions. In fact, any manipulation of load with any stopping rule (e.g., minimum completion, self-terminating with one of $n$ items being a target, exhaustive completion, etc.) is always relative. If it were possible to directly observe and compare serial versus parallel systems with each channel processing speed, the parallel systems would be revealed as less limited in capacity. However, as the present results demonstrate, simply an increase or decrease in RTs, even when the entire distribution is employed by the investigator, is by itself rather ambiguous.

In the serial exhaustive models implied in the proposition, the slower, nontarget processing times provide for a decrease in the sum of the probability terms from the single target-present trials. This slowdown can be sufficiently severe that the double-target probability term is actually larger, as the proof shows.

### *Plausibility Issues and Converging Conditions*

First, we discuss general aspects of model mimicking and then point out potentially beneficial converging conditions that help to ameliorate mimicking difficulties associated with the present result.

In many instances, model mimicking may be ameliorated by one of the models being unrealistic or nonintuitive. For instance, even though serial models can predict flat RT functions of an increasing load variable, even when processing must be exhaustive, they do it at the expense of having to be super capacity at an exceedingly unrealistic level. Therefore, such results were legitimately taken as supportive of parallel and rejective of serial processing (Townsend, 1974). On the other hand, limited-capacity parallel models could readily, and intuitively, predict the rising RT functions typically associated with serial processing in the face of increasing the load. Thus, an asymmetry in the impact of intuition and naturalness argumentation exists in this case. The consistent presence of super capacity is often a reliable indicator of there being something wrong with a process model because there is little in the cognition or human factors literature to suggest the presence of such an ability (Townsend & Colonius, in press; Townsend & Van Zandt, 1990).

Intuition is a slippery concept, and its force lies on a continuum. In the present matter, some may feel that for certain classes of experiments, such as those previously applying the race model inequality, that exhaustive serial processing is non-intuitive. However, the potential arena for the use of the race model inequality and the theoretical apparatus we have constructed is immense and inclusive of rather complex cognitive mechanisms. The parallel as well as serial (and probably hybrid) processes are likely to be found in random cognitive operations that may be open to identification employing the methodologies that we are developing. Suppose, for instance, a participant is given two sentences that differ in several ways but are semantically alike on a certain dimension. Subsequently, she or he is given a probe sentence that either does or does not match the target sentences on that semantic dimension with the instruction to push the "yes" button if either of the two sentences match the probe on the semantic dimension and the "no" button otherwise. Comparable studies have been carried out by J. R. Anderson and colleagues (Anderson, 1976). Although methodology derived from parallel–serial testability notions served to support parallel race models (Ross & Anderson, 1981), the question could surely not have been decided by intuition alone. This and many other cognitive purviews are readily adaptable to the race model inequality and associated methodology.

With regard to the exhaustive nature of the model, the likelihood that the nontarget channels are processed or searched when nontargets are simply blank stimuli may be small, such as in binocular detection when the nontarget channels are occupied by blank stimuli. Nevertheless, if a participant is rigid or conservative with regard to minimizing errors, the empty channels might well be inspected.

More critically, in a number of interesting contemporary experiments the choice is not between nothing and one or more targets, but between nontarget signals and target signals (Egeth & Dagenbach, 1991). In such tasks, there is even more of a possibility of exhaustive processing, as may take place, for example, in visual display or memory search tasks (Sternberg, 1966; cf. Van Zandt & Townsend, 1993). Because targets are often processed faster than nontargets (e.g., Bamber, 1969; Townsend & Roos, 1973), its imposition in the proposition is by no means unreasonable. Furthermore, it is easy to produce a corresponding theorem for self-terminating serial models, which are only slightly less nonintuitive than serial exhaustive models in the present context.

Physiological evidence may also be useful in ruling out theoretical alternatives, and some of these may indeed be in favor of parallelism or true coactivation rather than serial processing. The reader is referred to the discussion by Hughes, Reuter-Lorenz, Nozawa, and Fendrich (1994) of work of Stein, Meredith, and Wallace (1993) and others in this context. However, assuming behavioral cognitive psychology is to progress and interact with neuroscience in a mutually productive fashion, we need to know when various models and theories can mimic the other's operations.

Needless to say, cognitive and perceptual investigators typically do not want to include exhaustive serial models in their set of plausible alternatives, given violation of the race model inequality. The most appealing and realistic solution to the dilemma lies in the installation of converging conditions in reaching firm inferences concerning the process structure. For instance, it has been suggested that certain contingency effects, on the basis of, for instance, different probabilities of presenting two versus one target, may be problematic for coactive models but easy for specific types of parallel models (Mordkoff & Yantis, 1991). Another condition that can aid in rejecting exhaustive processing is to run the same redundant (and nonredundant) target conditions with a distractor rather than a blank stimulus in the nontarget position. If the cumulative distribution functions and redundancy gains are highly similar, then it is plausible that processing is self-terminating (parallel) in both cases. This tactic can be especially appealing if both sets of conditions are run on each participant, so that individual data may be analyzed.

Similarly, alternative dependent variables may certainly aid in model identification. For instance, the application of redundant targets designed with accuracy rather than RT as the major dependent variable is promising (Mordkoff & Egeth, 1993; Nozawa, 1992). Another example is the use of force in recent studies (Ulrich & Giray, 1986). It is beyond the scope of the present note to provide these developments or comments on the various solutions in detail.

However, our own research has dealt with the present and similar mimicking issues in redundant target designs and has assembled an experimental strategy, referred to as the *double factorial paradigm*, with accompanying theory and methodology (Townsend & Nozawa, 1995). We now briefly present the major ideas of that approach.

The central theme is to combine capacity analyses, including the race inequality, with systems factorial technology (Schweickert, 1978; Schweickert & Townsend, 1989; Schweickert & Wang, 1993; Townsend, 1984; Townsend & Ashby, 1983; Townsend & Schweickert, 1989). Systems factorial technology is a broad outgrowth of Sternberg's well-known additive factors method (Sternberg, 1969). In both, the assumption (testable to some extent; see Townsend & Thomas, 1994) that experimental factors selectively influence distinct subprocesses permits the identification of underlying architecture. The traditional analysis is based on the mean interaction contrast. This statistic, in a 2 $\times$ 2 design, computes the difference in mean RT at two different levels of one factor, combined with two different levels of another factor. For all levels of the factors, the load is constant (e.g., all examined trials are redundant target trials), so capacity expenditure is probably constant, certainly much more so than when the load (number of items) is varied.

In the double factorial paradigm (e.g., Townsend & Nozawa, 1995) the term *factor* is used in a somewhat different sense. The first factor is presence or absence of a target that may affect processing speed as seen above. The second factor is more arbitrary, but stimulus intensity is an example.

However, note that each of those factors (e.g., present vs. absent and bright vs. dim) is applied to each of two stimulus components, for example, two letters in a visual display, and therefore each of two channels. Hence, each channel provides a single typical 2 $\times$ 2 design. Perhaps *double-double factorial paradigm* might be a more apt name for the design! In principle, an experimenter could use four distinct factors, two on each channel, but presence versus absence of a target does play a central role in the current paradigm.

Recall that with manipulation of a single factor on two channels exhaustive serial models predict a null (i.e., mean interaction contrast of 0) interaction (Sternberg, 1969), whereas coactive models predict a positive interaction (i.e., overadditivity; Townsend & Nozawa, 1995). Exhaustive parallel models predict underadditivity. We have extended this approach to include tests at the distributional, rather than simply the mean, level of RTs and a number of other converging analyses.

In an application of the double factorial paradigm, Townsend and Nozawa (e.g., 1995) manipulated the presence or absence of a dot in each eye (Factor 1 of the double factorial paradigm) in addition to varying the brightness of the dot in either eye when a dot was present (Factor 2). This application to dot detection strongly falsified serial processing and exhaustive processing of either a serial or parallel variety. One experiment confirmed parallel race processing with mildly limited capacity, and the other supported coactive or interactive (mutually facilitory) parallel processing.[5]

[5] Additional evidence of the utility of the double factorial paradigm is starting to accrue (e.g., Nozawa, 1992).

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Received March 13, 1995
Revision received September 24, 1996
Accepted September 24, 1996 ■