British Journal of Mathematical and Statistical Psychology (1996), 49, 211–223 Printed in Great Britain

The relationship of variance to interaction contrast in parallel systems factorial technology

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Mean interaction contrast has been shown to be a beneficial statistic for testing between serial and parallel processes (e.g. Schweickert, 1978; Sternberg, 1969; Townsend, 1984). The present theoretical note investigates a conjecture of Egeth & Dagenbach (1991) that the mean interaction contrast is largest when the parallel processes are each deterministic, that is possessing zero variance. We show that their conjecture is true in certain situations but not in others. We further demonstrate that the magnitude of mean interaction contrast may be a non-monotonic function of processing time variance. Finally, implications for experimentation are pointed out.

1. Introduction

The investigation of simple mental architecture employing reaction time as a dependent variable reached a crescendo during the 1960s. Of particular interest was an important question dating back to the 19th century (e.g. Christie & Luce, 1956; Hamilton, 1859): Is the rapid searching of short-term memory or brief visual displays a parallel or serial operation (e.g. Sternberg, 1966; also see many references cited by Townsend & Ashby, 1983)? However, careful mathematical work was soon to demonstrate that within the common experimental paradigms, parallel and serial models were capable of mimicking one another's behaviour in a way that made testing the parallel–serial issue within those paradigms difficult to impossible (Atkinson, Holmgren & Juola, 1969; Townsend, 1969, 1972, 1976; Vorberg, 1977).

Over the intervening years, a number of experimental methods have appeared, mostly derived through mathematical exploration, capable of separating large classes of serial versus parallel models (summarized by Townsend, 1990a; see also Townsend & Ashby, 1983). One highly promising branch of research was evolved from the additive factor method proposed by Sternberg (1969). The idea was that two serially arranged subsystems (subprocesses, etc.), with no overlap in processing durations, would contribute in an additive fashion to the reaction time. Hence, if experimental factors could be found that selectively influence these separate

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subsystems, then the experimenter could predict a combined additive effect. That is, a zero factorial interaction should be found. If a significant interaction was discovered, then the proposed strategy was to conclude that selective influence failed, or that in actuality, only a single subsystem existed rather than two.

With regard to the method as originally put forth, detailed analyses, critiques, reviews and rigorous mathematical underpinnings have been contributed over the invervening years by a number of writers (e.g. Pachella, 1974; Pieters, 1983; Taylor, 1976; Theios, 1973; Townsend & Ashby, 1983, Chapter 12). About a decade later, investigators began to extend the theoretical framework under which factorial interactions, and the lack thereof, could pinpoint the type of mental architecture responsible for reaction time results in specific psychological tasks (Ashby, 1982; Ashby & Townsend, 1980; McClelland, 1979; Piotrowski, 1983; Schweickert, 1978, 1982, 1983; Townsend & Ashby, 1983; Townsend & Piotrowski, 1981). Schweickert and Townsend have recently developed a general stochastic theory of factorial interactions applicable to a wide variety of mental architectures describable as directed graphs (popularly known as PERT networks) with exhaustive processing at each separate node (Schweickert & Townsend, 1989; Townsend & Schweickert, 1989). The exhaustive assumption is weakened in recent developments by Nozawa (1989), Schweickert & Wang (1993), Townsend & Nozawa (1988) and Townsend & Nozawa (1995).

This growing body of knowledge has expanded the purview of the original methods so much that we have recently suggested the rubric 'systems factorial technology' to represent the present much enlarged theory and implied methodology as well as to encompass future developments (see e.g. Townsend & Nozawa, 1996; Townsend & Thomas, 1994). Fisher and Goldstein have also made important contributions to this enterprise (e.g. Fisher & Goldstein, 1983; Goldstein & Fisher, 1991, 1992). In the case of parallel processing, it turned out that parallel exhaustive processes predict an underadditive interaction whereas the minimum time of two parallel processes, conventionally referred to as a 'horse race' model, predicts an overadditive interaction (e.g. Egeth & Dagenbach, 1991; Schweickert & Wang, 1993; Townsend & Nozawa, 1988, 1996). Related theoretical work on parallel race models has been published by Colonius (1990) (see that paper and Townsend & Nozawa, 1995, for further citations).

Now, Egeth & Dagenbach (1991) suggest through intuitive reasoning that completely deterministic parallel processes will produce a maximum contrast interaction effect. That is, when any variance is attached to the parallel processes, the interaction contrast will be reduced. Of course, to keep matters 'fair,' the means for the various experimental conditions should be kept constant. If the hypothesis were true, it would also be of interest to inquire whether for any distributions and factorial effects leaving the means invariant, the interactions must monotonically decrease toward zero or some other limit as the variance continues to increase.

In the present note we prove that the first hypothesis is sometimes true, but only for certain 'speeds' of the separate channels. There are situations in which it fails. That is, in some cases, the interaction contrast is maximal when the variance is zero, but in others the contrast is larger for the stochastic model. Furthermore, there is a duality between the mean contrast of the minimum time random variable and that of the maximum time random variable (e.g. Townsend & Nozawa, 1995). Because of this duality it follows immediately that the absolute value of the mean contrast for the maximum is in the same direction as that for the minimum. Finally we show that there is no necessary relationship between the magnitude of the interaction contrast and the size of the variance. We will end the paper with remarks concerning the implications.

2. Theory

Consider a parallel model based on independent channels or processes, named $\mathbb{C}_x$ and $\mathbb{C}_y$. For instance, two parallel and independent counters, the first of which reaches a pre-set criterion setting off a detection response, would meet these specifications (e.g. Colonius, 1990; Townsend & Ashby, 1983, ch. 9). We assume that selective influence acts through the effect of an experimental factor ordering the cumulative distribution functions on processing times associated with each specific subprocess (Townsend, 1990*b*; Townsend & Ashby, 1983, pp. 280ff; Townsend & Schweickert, 1989). We assume the distribution functions are continuous on their domain of support, although that assumption could be easily weakened. The foregoing implies nonzero variance. We also wish to assume that the variances are finite.

It is often more convenient to use the survivor function, defined as one minus the distribution function, rather than the distribution function itself ($F(t)$). Obviously, an ordering in the distribution function causes the reverse ordering in the accompanying survivor functions. Let $S(t) = 1 - F(t)$ refer then, to the survivor function. Call the experimental factors X and Y and let one level of the factors be dubbed 's' for slow and the other 'f' for fast. Then by hypothesis, $S_{x\mathrm{f}}(t) \leq S_{x\mathrm{s}}(t)$ for all $t > 0$ and similarly for the distribution functions associated with channel $\mathbb{C}_y$ and factor Y. Also, assume that there exists an interval (a, b) where the inequality is strict in both cases.

It is also assumed that the processing on the two channels is stochastically independent. These assumptions then imply that the survivor function for the minimum (race winner) channel time for, say, the X = slow, Y = fast conditions, is just $S_{X\mathrm{s}}(t)S_{Y\mathrm{f}}(t)$ for any positive value of t. The mean (minimum) processing time is easily found to be

$$E(\mathbf{RT}; \mathrm{sf}; \min) = E[\min(\mathbf{T}_{X\mathrm{s}}, \mathbf{T}_{Y\mathrm{f}})] = \int_0^\infty S_{X\mathrm{s}}(t)S_{Y\mathrm{f}}\, \mathrm{d}t.$$

With the same assumptions the mean (maximum) processing time for the parallel exhaustive process is

$$E(\mathbf{RT}; \mathrm{sf}; \max) = E[\max(\mathbf{T}_{X\mathrm{s}}, \mathbf{T}_{Y\mathrm{f}})] = \int_0^\infty [1 - F_{X\mathrm{s}}(t)F_{Y\mathrm{f}}(t)]\, \mathrm{d}t$$

(see e.g. Luce, 1986 or Townsend & Ashby, 1983).

Letting **RT** be the reaction time random variable and shortening, say, $E(\mathbf{RT}; \mathrm{sf})$ to M_{sf} for the minimum and N_{sf} for the maximum, the mean interaction contrast for the minimum is defined as

$$\mathrm{C}_{X,Y}^{(\min)} = \mathrm{M}_{\mathrm{ss}} - \mathrm{M}_{\mathrm{sf}} - \mathrm{M}_{\mathrm{fs}} + \mathrm{M}_{\mathrm{ff}},$$

whereas ss refers to the X = slow, Y = slow condition and so on (e.g. Townsend, 1984; Townsend & Ashby, 1983, Chapter 12; Dzhafarov, 1993). The formula for the maximum is obvious. Now, it is typically necessary to append a residual (early sensory, motor, etc.) time random variable to the processing time random variable to produce the overall reaction time. However, if, as is usually assumed, the residual variable is independent and unaffected by the processing time variables, the above contrast formula can be rewritten in terms of the parallel processing time variables, convolved with the residual term (see Townsend & Nozawa, 1988, 1995, for a substantial generalization and Dzhafarov, 1992 for an alternative conception of residual times). Therefore, the residual variable may be neglected without affecting the generality of the present results.

Under the postulated conditions, we see that the mean interaction contrast for the race model is

$$C^{(\min)}_{(X,Y)} = M_{ss} - M_{sf} - M_{fs} + M_{ff}$$
$$= \int_0^\infty S_{Xs}(t)S_{Ys}(t)\,dt - \int_0^\infty S_{Xs}(t)S_{Yf}(t)\,dt - \int_0^\infty S_{Xf}(t)S_{Ys}(t)\,dt + \int_0^\infty S_{Xf}(t)S_{Yf}(t)\,dt$$
$$= \int_0^\infty [S_{Xs}(t) - S_{Xf}(t)][S_{Ys}(t) - S_{Yf}(t)]\,dt, \quad (1)$$

and for the exhaustive model

$$C^{(\max)}_{(X,Y)} = N_{ss} - N_{sf} - N_{fs} + N_{ff}$$
$$= \int_0^\infty [1 - F_{Xs}(t)F_{Ys}(t)]\,dt - \int_0^\infty [1 - F_{Xs}(t)F_{Yf}(t)]\,dt$$
$$- \int_0^\infty [1 - F_{Xf}(t)F_{Ys}(t)]\,dt + \int_0^\infty [1 - F_{Xf}(t)F_{Yf}(t)]\,dt$$
$$= \int_0^\infty [S_{Xf}(t) - S_{Xs}(t)][S_{Ys}(t) - S_{Yf}(t)]\,dt. \quad (2)$$

Notice that the contrast in the latter case is just the negative of that in the former. This is a version of the duality mentioned earlier. How shall we compare this result with the deterministic case, where the variances are all zero? To be interesting, the means should be kept constant, as suggested by Egeth & Dagenbach (1991). That is, the means in the stochastic case (non-zero variance) should be the same as the deterministic times (zero variance). Write the stochastic (non-degenerate) *individual* means t_{ij}, where i is X or Y and j is the level of the factor. Now, the mean finishing time for the minimum time model in the deterministic case is the time of the faster channel under all conditions. Clearly, which time enters into the deterministic formula for contrast depends on the relative ordering of the fixed times. The derivatives differ trivially depending on that order, so we perform only one in detail. We will prove it with the ordering associated with Fig. 1, with the other cases being handled in like manner (see below for an exact description for the model associated with Fig. 1). Let $t_{(Y,f)} < t_{(X,f)} < t_{(X,s)} < t_{(Y,s)}$. It is then easily seen that the minimum time contrast in the deterministic race scenario can be expressed as

$$C^{(\min)}_{X,Y} = M_{ss} - M_{sf} - M_{fs} + M_{ff}$$
$$= t_{X,s} - t_{Y,f} - t_{X,f} + t_{Y,f}$$
$$= t_{X,s} - t_{X,f} \geq 0.$$

For the parallel exhaustive process the contrast can be written as

$$C^{(\max)}_{X,Y} = N_{ss} - N_{sf} - N_{fs} + N_{ff}$$
$$= t_{Y,s} - t_{X,s} - t_{Y,s} + t_{X,f}$$
$$= t_{X,f} - t_{X,s} \leq 0$$

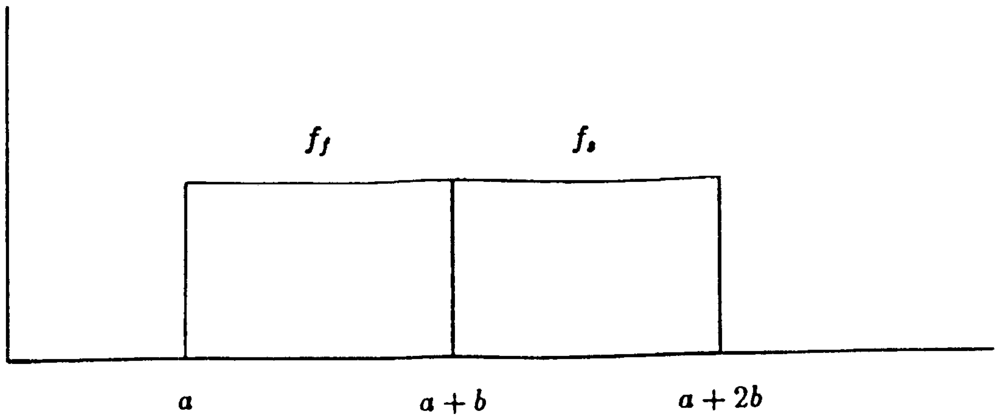

Figure 1. Uniform distributions with means $\mu_f = \frac{1}{2}(2a+b)$, $\mu_s = \frac{1}{2}(2a+3b)$ and variances $\sigma^2 = b^2/12$.

Notice that, as expected, the contrast for the maximum is again, just as in the stochastic case, the negative of that for the minimum.

We are now in a position to show that the contrast for the nondegenerate stochastic model is greater than that for the deterministic model *for this ordering*. The key to the proof is to note that the deterministic times, for example $t_{(X,s)}$ and $t_{(X,f)}$ can be written in terms of the stochastic distributions, since the quantity represented by the mean is not changed, and recalling that the individual channel means of the stochastic case (which we notate as μ) must be equal to the fixed individual 'means' of the deterministic case. Hence $t_{(X,s)} = \mu_{(X,s)}$, $t_{(X,f)} = \mu_{(X,f)}$, whereas remembering that the means can be expressed as an integral of the survivor functions we find that

$$\mu_{(X,s)} = \int_0^\infty S_{Xs}(t)\, dt$$

and so on. Thus, with

$$t_{(X,s)} - t_{(X,f)} = \int_0^\infty [S_{Xs}(t) - S_{Xf}(t)]\, dt$$

the appropriate comparison for the race model leads to the inequality

$$0 \le \int_0^\infty [S_{Xs}(t) - S_{Xf}(t)][S_{Ys}(t) - S_{Yf}(t)]\, dt \le \int_0^\infty [S_{Xs}(t) - S_{Xf}(t)]\, dt,$$

and for the exhaustive model leads to the inequality

$$0 \ge \int_0^\infty [S_{Xf}(t) - S_{Xs}(t)][S_{Ys}(t) - S_{Yf}(t)]\, dt \ge \int_0^\infty [S_{Xf}(t) - S_{Xs}(t)]\, dt,$$

since letting $0 \le S_{Ys}(t) - S_{Yf}(t) \le 1$ for all $t \ge 0$ the stochastic side is always less than or equal to the deterministic side in absolute value. This proves part (A) of Proposition 2.1, below.

As the reader may ascertain, there are only two other nontrivial permutations to investigate. The first is $t_{(Y,f)} < t_{(X,f)} < t_{(Y,s)} < t_{(X,s)}$ and the second is $t_{(Y,f)} < t_{(Y,s)} < t_{(X,f)} < t_{(X,s)}$. Using similar procedures as those described above we may complete the proofs for Proposition 2.1.

Keep in mind that what holds for the contrast on minimum time holds for the maximum time in absolute value. Hence, we provide proofs only for the minimum cases.

Proposition 2.1 The relationship between the stochastic versus deterministic models is given for each of the three orders as follows:

(A) $t_{(Y,\mathrm{f})} < t_{(X,\mathrm{f})} < t_{(X,\mathrm{s})} < t_{(Y,\mathrm{s})}$: *Absolute value of the deterministic contrast > stochastic contrast.*
(B) $t_{(Y,\mathrm{f})} < t_{(X,\mathrm{f})} < t_{(Y,\mathrm{s})} < t_{(X,\mathrm{s})}$: *The order of the (absolute value of the) stochastic versus deterministic contrast depends on the particulars.*
(C) $t_{(Y,\mathrm{f})} < t_{(Y,\mathrm{s})} < t_{(X,\mathrm{f})} < t_{(X,\mathrm{s})}$: *Absolute value of the stochastic contrast > determinstic contrast* $= 0$.

Proof. (A) Completed in text.

(B) Case (i): Deterministic contrast is larger in absolute value than stochastic contrast. Let all distributions be exponential: $\lambda_i e^{-\lambda_i t}$ where $\lambda_i = X_i$ or Y_i and $i =$ s or f. The comparison is then

$$\frac{1}{X_s + Y_s} - \frac{1}{X_s + Y_f} - \frac{1}{X_f + Y_s} + \frac{1}{X_f + Y_f} \text{ vs } \frac{1}{Y_s} - \frac{1}{X_f}.$$

Now let $Y_f = 1000$, $X_f = 900$, $Y_s = 100$, $X_s = 99$, to prove (i). Case (ii): Stochastic is larger in absolute value than deterministic. Again use exponential model, but now with parameters $Y_f = 1000$, $X_f = 500$, $Y_s = 450$, $X_2 = 1$ to reverse the inequality.

(C) From equation (1) it is apparent that the stochastic side is always greater than 0. However, it is equally clear that the deterministic contrast from the minimum time or the maximum time is simply 0. □

A related question ensuing from the Egeth & Dagenbach (1991) conjecture is whether the mean contrast must be a monotonic function of variance. This is not so, as expressed in Proposition 2.2.

Proposition 2.2. There exist probability distributions whose associated contrast functions are nonmonotonic functions of variance.

Proof. Consider, as one example, uniform distributions with means μ_f and μ_s for the fast condition and the slow condition, respectively, with $\mu_f < \mu_s$ and variances σ^2 for both conditions. More concretely, let the uniform distribution for the fast condition be defined on the closed interval $[a, a + b]$, $a \geq 0$, $b > 0$, i.e. with density function

$$f_f = \begin{cases} 0 & 0 < t < a \\ \frac{1}{b} & a \leq t \leq a + b \\ 0 & a + b < t, \end{cases}$$

mean $\mu_f = \frac{1}{2}(2a + b)$ and variance $\sigma^2 = b^2/12$. The uniform distribution for the slow condition is defined

$$f_s = \begin{cases} 0 & 0 < t < a + b \\ \frac{1}{b} & a + b \leq t \leq a + 2b \\ 0 & a + 2b < t \end{cases}$$

with mean $\mu_s = \frac{1}{2}(2a + 3b)$ and variance $\sigma^2 = b^2/12$. Figure 1 shows this situation. Since the contrast is defined as $C_p = \int (S_s(t) - S_f(t))^2 \, dt$ we need to determine the survivor functions S_f and S_s for the fast and slow conditions, respectively; that is

$$1 - F_f = S_f = \begin{cases} 1 & 0 < t < a \\ 1 - \frac{t - a}{b} & a \leq t \leq a + b \\ 0 & a + b < t \end{cases}$$

and

$$1 - F_s = S_s = \begin{cases} 1 & 0 < t < a + b \\ 1 - \frac{t - a - b}{b} & a + b \leq t \leq a + 2b. \\ 0 & a + 2b < t \end{cases}$$

According to the different intervals on which the distributions are defined, we determine the contrast piecewise, where $C_p = C_{p1} + C_{p2} + C_{p3}$ and the indices 1, 2 and 3 refer to different intervals, i.e.

$$C_{p1} = \int_0^a (1 - 1)^2 \, dt = 0 \qquad 0 < t < a$$

$$C_{p2} = \int_a^{a+b} \left[1 - \left(1 - \frac{t - a}{b}\right)\right]^2 dt = \frac{b}{3} \qquad a \leq t \leq a + b$$

$$C_{p3} = \int_{a+b}^{a+2b} \left[1 - \frac{t - a - b}{b}\right]^2 dt = \frac{b}{3} \qquad a + b \leq t \leq a + 2b$$

and

$$C_p = C_{p1} + C_{p2} + C_{p3} = \frac{2}{3}b \qquad 0 \leq t \leq a + 2b. \tag{3}$$

Consider now the case where we have again uniform distributions for the fast and slow conditions with exactly the same means as before but with a smaller variance for both conditions, i.e. the uniform distribution for the fast condition is now defined on the closed interval $[a + \frac{1}{2}a, a - \frac{1}{2}a + b]$ and the uniform distribution for the slow condition is defined on the interval $[a + \frac{1}{2}a + b, a - \frac{1}{2}a + 2b]$, $b > a$ with respective density functions

$$f_f = \begin{cases} 0 & 0 < t < a + \frac{1}{2}a \\ \frac{1}{b - a} & a + \frac{1}{2}a \leq t \leq a - \frac{1}{2}a + b \\ 0 & a - \frac{1}{2}a + b < t \end{cases}$$

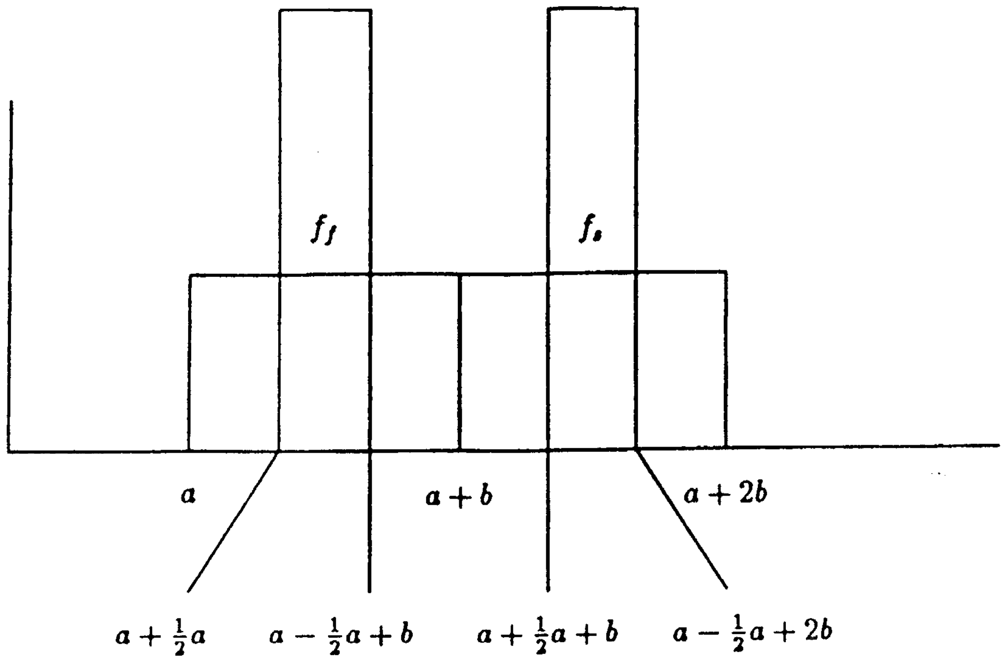

Figure 2. Uniform distributions with means $\mu_f = \frac{1}{2}(2a+b)$, $\mu_s = \frac{1}{2}(2a+3b)$ and variances $\sigma^2 = (b-a)^2/12$.

and

$$f_s = \begin{cases} 0 & 0 < t < a + \frac{1}{2}a + b \\ \dfrac{1}{b-a} & a + \frac{1}{2}a + b \le t \le a - \frac{1}{2}a + 2b \\ 0 & a - \frac{1}{2}a + 2b < t. \end{cases}$$

Figure 2 illustrates these densities. The survivor functions for both conditions in this situation are, respectively

$$1 - F_f = S_f = \begin{cases} 1 & 0 < t < a + \frac{1}{2}a \\ 1 - \dfrac{t - a - \frac{1}{2}a}{b-a} & a + \frac{1}{2}a \le t \le a - \frac{1}{2}a + b \\ 0 & a - \frac{1}{2}a + b < t \end{cases}$$

and

$$1 - F_s = S_s = \begin{cases} 1 & 0 < t < a + \frac{1}{2}a + b \\ 1 - \dfrac{t - a - \frac{1}{2}a - b}{b-a} & a + \frac{1}{2}a + b \le t \le a - \frac{1}{2}a + 2b \\ 0 & a - \frac{1}{2}a + 2b < t. \end{cases}$$

Again the contrast is split up into three parts:

$$C_{p1} = \int_0^{a+\frac{1}{2}a} (1 - 1)^2 \, dt = 0 \qquad 0 < t < a + \frac{1}{2}a$$

$$C_{p2} = \int_{a+\frac{1}{2}a}^{a-\frac{1}{2}a+b} \left[1 - \left(1 - \frac{t - a - \frac{1}{2}a}{b - a}\right)\right]^2 dt = \frac{b - a}{3} \qquad a + \frac{1}{2}a \le t \le a - \frac{1}{2}a + b$$

$$C_{p3} = \int_{a+\frac{1}{2}a+b}^{a-\frac{1}{2}a+2b} \left[1 - \frac{t - a - \frac{1}{2}a - b}{b - a}\right]^2 dt = \frac{b - a}{3} \qquad a + \frac{1}{2}a + b \le t \le a - \frac{1}{2}a + 2b.$$

The contrast for the entire domain is

$$C_p = C_{p1} + C_{p2} + C_{p3} = \frac{2}{3}(b - a) \qquad 0 \le t \le a - \frac{1}{2}a + 2b. \tag{4}$$

Comparing equations (3) (4), $\frac{2}{3}(b - a) < \frac{2}{3}b$ shows that the contrast *decreases* when the variance decreases, the means remaining the same in both situations.

Finally consider the situation where we have uniform distributions for the fast and slow conditions with the same means as before but now with a large variance for both conditions, i.e. the uniform distribution for the fast condition is now defined on the closed interval $[0, 2a + b]$ and the uniform distribution for the slow condition is defined on the closed interval $[b, 2a + 2b]$. The density distributions in this case are

$$f_f = \begin{cases} 0 & t < 0 \\ \dfrac{1}{2a + b} & 0 \le t \le 2a + b \\ 0 & t > 2a + b \end{cases}$$

and

$$f_s = \begin{cases} 0 & 0 < t < b \\ \dfrac{1}{2a + b} & b \le t \le 2a + 2b \\ 0 & t > 2a + 2b \end{cases}$$

both having variances $\sigma^2 = \dfrac{(2a + b)^2}{12}$. Figure 3 shows this situation compared with the first one.

For the respective survivor functions we get

$$1 - F_f = S_f = \begin{cases} 1 & 0 < t \\ 1 - \dfrac{t}{2a + b} & 0 \le t \le 2a + b \\ 0 & t > 2a + b \end{cases}$$

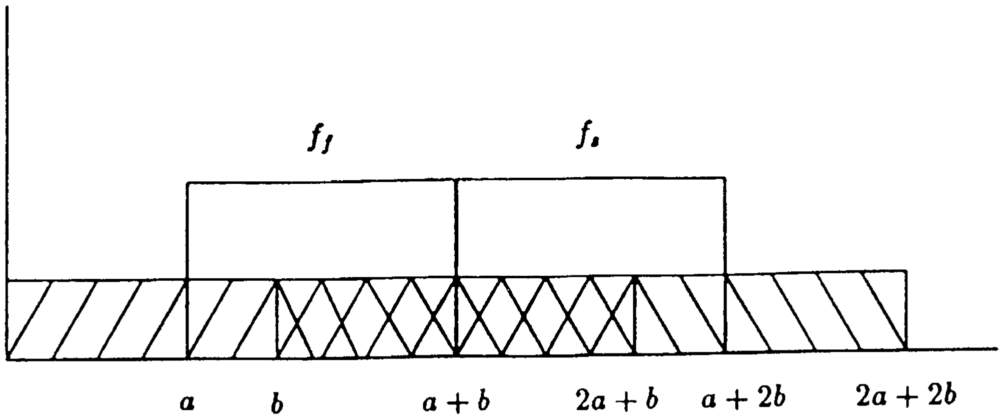

Figure 3. Uniform distributions with means $\mu_f = \frac{1}{2}(2a+b)$, $\mu_s = \frac{1}{2}(2a+3b)$ and variance $\sigma^2 = (2a+b)^2/12$.

and

$$1 - F_s = S_s = \begin{cases} 1 & 0 < t < b \\ 1 - \dfrac{t-b}{2a+b} & b \le t \le 2a + 2b \\ 0 & t > 2a + 2b. \end{cases}$$

The contrast is

$$C_p = C_{p1} + C_{p2} + C_3 = \frac{2b^2(3a+b)}{3(2a+b)^2} \qquad 0 \le t \le 2a+2b \tag{5}$$

with

$$C_{p1} = \int_0^b \left[1 - \left(1 - \frac{t}{2a+b}\right)\right]^2 \mathrm{d}t = \frac{b^3}{3(2a+b)^2} \qquad 0 < t < b$$

$$C_{p2} = \int_b^{2a+b} \left[1 - \frac{t}{2a+b} - \left(1 - \frac{t-b}{2a+b}\right)\right]^2 \mathrm{d}t = \frac{2ab^2}{(2a+b)^2} \qquad b \le t \le 2a+b$$

$$C_{p3} = \int_{2a+b}^{2a+2b} \left[1 - \frac{t-b}{2a+b}\right]^2 \mathrm{d}t = \frac{b^3}{3(2a+b)^2} \qquad 2a+b \le t \le 2a+2b.$$

Comparing equations (3) and (5)

$$\frac{2}{3}\frac{b^2(3a+b)}{(2a+b)^2} < \frac{2}{3}b$$

$$\frac{b(3a+b)}{(2a+b)^2} < 1$$

$$0 < 4a^2 + ab \qquad a, b > 0$$

shows that the contrast *increases* as the variance decreases. □

3. Discussion

The present results are closely related to the experimental detection of factor interaction and thus have a direct impact in the model testing arena.

Proposition 2.1 demonstrates that the Egeth & Dagenbach (1991) conjecture, that the presence of variance always diminishes mean interaction contrast, is not universally true. It holds for some orders of processing times and not for others. Proposition 2.2 indicates that the size of mean contrast also may not be monotonically related to magnitude of variance.

Another point, emphasized by inspection of equations (1) and (2) is that the levels of experimental factors should be selected so that the distributions are separated as far apart as feasible. Obviously, a somewhat separate point from the main goal of this paper, is that extraneous variance should always be minimized, relative to the numerical mean interaction contrast, in order to increase statistical power (cf. Townsend, 1984).

Acknowledgements

This research was partly supported by the National Science Foundation Grant no. 9112813, and a James McKeen Cattell Sabbatical Award to the first author. The collaborative effort was made possible by a grant from the German American Commission on Collaborative Research sponsored by the American ACLS and the German DAAD. We thank Hans Colonius, Pietro Michelucci and Jerry Balakrishnan for their comments, two referees for their anonymous critiques, and Sheryl Mobley and Monika Niemann for preparing the manuscript.

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Received 29 December 1993; revised version received 26 September 1995