4 Dynamic Representation of Decision-Making

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EDITORS' INTRODUCTION

Those unfamiliar with the dynamical approach often suspect that, while it might be appropriate for low-level or peripheral aspects of cognition, it cannot be used to describe high-level or central aspects. Yet nothing could be more central—more paradigmatically cognitive—*than processes of decision-making, the target of Townsend and Busemeyer's work. Their decision field theory (DFT), described in this chapter, is a general dynamical and stochastic framework for modeling decision-making which accounts for data covered by traditional, static-deterministic theories, but whose explanatory capacities go beyond those of traditional theories in a number of respects.*

Virtually all quantitative theories of decision-making in psychology, and in cognitive science more generally, are versions of subjective expected utility theory. Many beautiful mathematical theorems have been established that, at the very least, serve as useful guides to optimal choices. Yet, for the great majority of theories and applications to empirical phenomena, there have been no explicit psychological dynamics whatsoever—that is, no attempt to trace out the actual mental processes the subject goes through in reaching a decision. The modus operandi *has simply been to compare two potential choices (i.e., gambles, etc.) and conclude that the decision-maker should choose the one with the higher expected utility. When inevitable "paradoxes" appear, in which human decision-makers do not behave as the theory proclaims, the standard response is to alter the axioms (e.g., change the form of the utility function).*

The DFT framework, by contrast, sets out with the explicit aim of modeling the psychological processes involved in decision-making. In this framework the system begins in a certain preference state with regard to certain choices, and this state evolves over time according to dynamical equations which govern the relationship among factors such as the motivational value of an outcome and the momentary anticipated value of making a particular choice. Importantly, DFT models are able to account for the standard psychological data on the kinds of choices people make, and indeed predict certain data that appear paradoxical from the traditional perspective.

Since key variables in a DFT model evolve over time, the model builds in the capacity to account for temporal features of the deliberation process, such as the way *the decision a subject makes depends on deliberation time. Such temporal considerations are inherently out of the reach of traditional models which are either entirely static or specify at best just a bare sequence of steps. The DFT framework thus provides a powerful illustration of the explanatory advantages of adopting a dynamical framework which supposes from the outset that cognitive processes essentially evolve over real time.*

4.1 INTRODUCTION

The deliberation may last for weeks or months, occupying at intervals the mind. The motives which yesterday seemed full of urgency and blood and life to-day feel strangely weak and pale and dead. But as little to-day as to-morrow is the question finally resolved. Something tells us that all this is provisional; that the weakened reasons will wax strong again, and the stronger weaken; that equilibrium is unreached; that testing our reasons, not obeying them, is still the order of the day, and that we must wait awhile, patient or impatiently, until our mind is made up "for good and all." This inclining first to one then to another future, both of which we represent as possible, resembles the oscillations to and fro of a material body within the limits of its elasticity. There is inward strain, but no outward rapture. And this condition, plainly enough, is susceptible of indefinite continuance, as well in the physical mass as in the mind. If the elasticity give way, however, if the dam ever do break, and the current burst the crust, vacillation is over and decision is irrevocably there.

—William James, *The Principles of Psychology* (1890/1950, p. 529.).

This deliberation process, so eloquently described by William James more than 100 years ago, seems to be engaged whenever we are confronted with serious decisions such as getting married or divorced, having a child, quitting a job, undergoing elective surgery, or other life-threatening decisions. This process still occurs, but to a lesser extent, with more commonplace decisions such as choosing a car, buying a computer, or planning a vacation. The process is manifested by indecisiveness, vacillation, inconsistency, lengthy deliberation, and distress (Janis and Mann, 1977; Svenson, 1992).

It seems odd that many psychological theories of decision-making fail to mention anything about this deliberation process. Many previous theories of decision-making (e.g., the prospect theory of Kahneman and Tversky, 1979) assume that for any particular situation, individuals assign weights and values to each possible outcome, and the final decision is simply a matter of comparing the summed products of weights and values for each alternative. The entire process is described in a deterministic and static manner. There is no explanation for changes in state of preference over time, and there is no mechanism for deriving the time needed for deliberation. This criticism applies equally well to all static-deterministic theories of risky decision making that have evolved from the basic expected utility formulation (von Neumann and Morgenstern, 1947; Savage, 1954).

We are not claiming that static theories are irrelevant to the understanding of human decision-making. On the contrary, ideas from these theories can be incorporated into the present framework. Instead, we claim that these static theories are seriously incomplete owing to their failure to explain the psycho-

Table 4.1 Decision theory taxonomy

	Static	Dynamical
Deterministic	Expected utility	Affective balance
Probabilistic	Thurstone utility	Decision field theory

Note: Thurstone's utility theory is an example of a more general class called random utility theories (Thurstone, 1959). Affective balance theory was proposed by Grossberg and Gutowski (1987).

logically important dynamical phenomena of human conflict—the evolution of preferences over time during conflict resolution.

The new contribution of decision field theory can be characterized by considering table 4.1, which provides a classification of theories according to two attributes—"deterministic vs. probabilistic," and "static vs. dynamic." *Deterministic* theories postulate a binary preference relation which is either *true or false* for any pair of actions. *Probabilistic* theories postulate a probability function that maps each pair of actions into the closed interval [0,1]. *Static* theories assume that the preference relation (for deterministic models) or the probability function (for probabilistic models) is independent of the length of deliberation time. *Dynamical* theories specify how the preference relation or probability function changes as a function of deliberation time. For the past 45 years, the *deterministic-static* category has dominated research on decision-making under uncertainty. Decision field theory builds on this past work by extending these theories into the *stochastic-dynamical* category.

The purpose of this chapter is to provide a general overview of an alternative framework for understanding decision-making called decision field theory (DFT). Decision field theory provides a dynamical, stochastic description of the *deliberation process* involved in decision-making. It is unique in its capability for deriving *precise* quantitative predictions for (a) the probability of choosing each alternative as a function of deliberation time (Busemeyer and Townsend, 1993), (b) the mean deliberation time needed to make a decision (Busemeyer and Townsend, 1993), (c) the distribution of selling prices, buying prices, and certainty equivalents for gambles (Busemeyer and Goldstein, 1992), and (d) approach-avoidance movement behavior (Townsend and Busemeyer, 1989).

Decision field theory is based on psychological principles drawn from three different areas of psychology. The first is the early learning and motivation theories of approach-avoidance conflict developed by Lewin (1935), Hull (1938), and Miller (1944). The second is the more recent information-processing theories of choice response time (see Townsend and Ashby, 1983; Luce, 1986). The third is research and theory on human decision-making, especially the recent work by Coombs and Avrunin (1988).

The remainder of this chapter is organized as follows. The basic assumptions of DFT are summarized in section 4.2. A brief review of how the theory is applied to choice and selling price preference tasks is presented in section 4.3. Then, in section 4.4, DFT is used to explain two different "paradoxical" empirical findings. The main message of this chapter is the following: often what appears to be "paradoxical" behavior from the viewpoint of static-deterministic theories turns out to be emergent properties of the dynamical-stochastic nature of the human deliberation process.

4.2 GENERAL THEORETICAL STRUCTURE

Figure 4.1 provides an outline of DFT. On the far left are the values of all the potential consequences produced by each course of action. In the figure, six consequences are shown: three rewards or gains, and three punishments or losses. A distinction is made between rewards or attractive consequences, and punishments or aversive consequences. The values of the six consequences can be organized into a 6- × 1-vector **M**, where the first three elements contain the values of the three gains (forming a 3- × 1-subvector

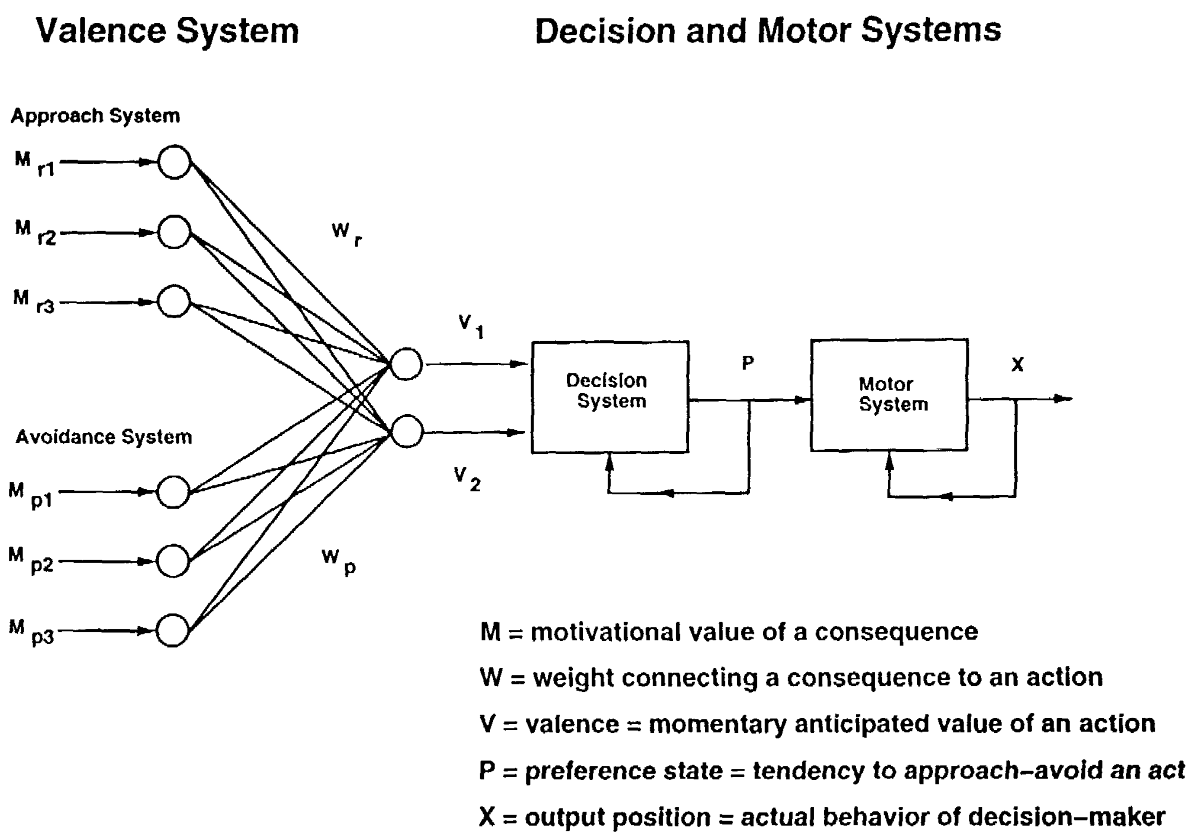


Figure 4.1 Diagram of DFT. At the far left are the gains and losses produced by each course of action, denoted *M*. These inputs are then filtered by a set of attention weights that connect each action to each consequence, denoted *W*. These filtered values form the valence or momentary anticipated value of each action, denoted *V*. Valence corresponds to force in a physical system. Then the valence is input to a decision system which temporally integrates the valences to produce a preference state for each action as an output, denoted *P*. Preference state corresponds to velocity in a physical system. Finally, the preference is input to a motor system which temporally integrates the preferences over time to produce the observed action, denoted *X*. The observed action corresponds to the physical position in a physical system.

M_r) and the last three elements contain the values of the three losses (forming another 3- × 1-subvector M_p).

Each course of action has some connection or association with each consequence. The strength of an individual's belief in the connection between each consequence and each action is represented by a weight $W_{ij}(t)$. In the figure, two acts lead to six possible consequences, producing a total of 12 connection weights. These 12 connection weights can be organized into a 2-×6-weight matrix symbolized as $\mathbf{W}(t) = [\mathbf{W}_r(t)|\mathbf{W}_p(t)]$, where $\mathbf{W}_r(t)$ is the 2-×3-submatrix of weights for the rewards, and $\mathbf{W}_p(t)$ is the 2-×3-submatrix of weights for the punishments.

The connection weights fluctuate over time during deliberation, reflecting changes in the decision-maker's attention to the various consequences produced by each action. For example, at one moment the decision-maker may think about the gains produced by choosing an action, but at a later moment the decision-maker's attention may shift toward the potential losses.

The set of connection weights act like a filter that modifies the impact of the input values of the consequences **M**. The output of this filtering process is a vector called the valence vector, denoted $\mathbf{V}(t)$. In the figure, $\mathbf{V}(t)$ is a 2-×1-vector because only two actions ($V_1(t)$ and $V_2(t)$) are shown in the figure. Each element of this valence vector represents the momentary anticipated value that would be produced by choosing a particular course of action.

The valence is transformed into action by passing through two different dynamical systems—a decision system and then a motor system. First, valence is the input for a decision system that produces a preference state as output, representing the vector $\mathbf{P}(t)$. In the figure, there are only two actions so that $\mathbf{P}(t)$ has two elements, $P_1(t)$ and $P_2(t)$. Each coordinate of $\mathbf{P}(t)$ represents the temporal integration of the valences generated by a course of action. Thus, each coordinate of $\mathbf{P}(t)$ represents the current estimate of the strength of preference for a course of action.

Finally, the preference state becomes the input for a motor system that produces a response or overt movement as output. The physical position at each time point of the motor mechanism used to execute an action is represented by a vector $\mathrm{X}(t)$.

The basic concepts of DFT shown in figure 4.1 are described in more detail below, beginning with the observable end product (actions) and working backward to the unobservable driving force (valence).

Motor System

In general, the preference state is the input into a motor system that produces a movement as output. If $\mathrm{X}(t)$ is the physical position at time t, and $\mathrm{X}(t + h)$ is the physical position at the next instant in time, then $d\mathrm{X}(t + h)) = \mathrm{X}(t + h) - \mathrm{X}(t)$ is the change in physical position during the small time interval h. The velocity of the movement, denoted $d\mathrm{X}(t + h)/h$, is represented by a difference equation:

$$dX(t + h)/h = R[X(t), P(t + h)]. \tag{1}$$

In other words, the velocity of the movement is a function of the previous position and the current preference state. The physical position $X(t)$ is the integration of the velocity over time. The detailed specification of the response function R depends on the nature of the movement. Later in this chapter, we consider two different types of responses commonly used to measure preference: choice and selling prices.

It is crucial to note that only the motor system changes when different response measures are used for measuring preference. The valence system and the decision system remain invariant. This provides strong leverage for testing the theory; the parameters of the theory may be estimated using one response measure, and then these same parameter values are used to make parameter-free predictions for the remaining response measures. This cross-validation method for testing the model is illustrated later in this chapter.

Decision System

The preference state is driven by the incoming valence associated with each act (see figure 4.1). The valence at time t, denoted $\mathbf{V}(t)$, is a point within the preference space that pushes or pulls the preference state at time t. The force of the valence on the preference state is represented by a linear *difference* equation:

$$d\mathbf{P}(t + h)/h = -\mathbf{S} \cdot \mathbf{P}(t) + \mathbf{C} \cdot \mathbf{V}(t + h) \tag{2}$$

In other words, the rate and direction of change in preference is a linear function of the previous preference state and the incoming valence. The preference state $\mathbf{P}(t)$ is the integration of these forces over time.

The constant matrix $\mathbf{S}$ is called the stability matrix, and it controls the rate of growth of preferences. This is similar to a learning rate parameter in a linear operator learning model (cf. Bush and Mosteller, 1955). For example, if a constant positive valence is applied to one act, then the preference for that act gradually increases from the initial zero state toward the constant value. If the valence is later reset to zero, then preference gradually decays from the previous asymptote toward zero. The rate of growth and decay is determined by the stability matrix $\mathbf{S}$.

The constant matrix $\mathbf{C}$ is called the contrast matrix, and it determines how acts are compared to form preferences. To see how this works, assume that there are three acts. If all three acts are evaluated independently, then $\mathbf{C} = \mathbf{I}$, the identity matrix. In this case, preference for all three actions may increase simultaneously, producing a "race" toward each goal.

Alternatively, one action may be compared to the average of the remaining two

$$\mathbf{C} = \begin{matrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2. \\ -1/2 & -1/2 & 1 \end{matrix}$$

In the above case, increasing the preference for one alternative corresponds to a decrease in preference for the remaining two alternatives. This would be appropriate if movement toward one goal entailed movement away from other goals.

The linear form of the above difference equation was chosen for four reasons. First, it is the simplest form capable of generating the desired type of behavior. Second, it is mathematically tractable, which allows derivation of interesting empirical tests. Third, it reduces as a special case to a number of previously developed models of decision-making. Fourth, the linear form may be considered a rough approximation to some nonlinear form. Linear approximations have proved to be useful in physics and engineering for analyzing problems within a limited domain. Hopefully, the failures of the linear model will indicate the type of nonlinearity needed to provide a more accurate representation of the dynamics. Townsend and Busemeyer (1989) began a probe of nonlinear dynamics within DFT.

Valence System

Valence is the motivational source of all movement. It is determined by two factors (see figure 4.1): (a) an n- $\times$ m-weight matrix, $\mathbf{W}(t)$, representing the strength of connection between each act and consequence, and (b) an m- $\times$ l-column vector, $\mathbf{M}(t)$, representing the motivational values of each consequence. *Valence* is defined as the matrix product of the weight matrix and the motivational value vector,

$$\mathbf{V}(t) = \mathbf{W}(t) \cdot \mathbf{M}(t). \tag{3}$$

Each element, $V_i(t)$, is a weighted sum of motivational values.

The n- $\times$ m-weight matrix, denoted $\mathbf{W}(t)$, represents the moment-to-moment strength of connection between each act and each consequence. An act-consequence connection refers to the expectation that, under a given set of environmental conditions, an act produces a relevant consequence at some later point in time. The weight $W_{ij}(t)$ connecting act i to consequence j at time t ranges from zero to unity; $0 \leqslant W_{ij}(t) \leqslant 1$; $W_{ij}(t) = 0$ means that the motivational value of consequence j has no influence on act i; $W_{ij}(t) = .5$ means that the motivational value of consequence j is reduced by one half for act i; $W_{ij}(t) = 1$ means that the full force of the motivational value of consequence j is applied to act i. These weights are determined by the product of six factors: attention, learning, relevance, probability, temporal distance, and physical distance.

Attention to an act-consequence connection means that the connection has been retrieved from long-term memory and it is active in short-term memory. Models of memory retrieval (e.g., Raaijmakers and Shiffrin, 1981) may be useful for predicting the effects of attention on decision-making.

Learning refers to changes in the strength of an act-consequence connection based on experience with previous decisions or instruction. Models of learning (e.g., Busemeyer and Myung, 1992) may be used to describe changes in connection strength resulting from experience.

When the weights represent the temporal remoteness of the consequences, then the valence of each act is equivalent to a temporal discounting model (see Stevenson, 1986) *at a specific moment*.

When the weights represent the probabilities of the consequences, then the valence of each act is equivalent to a subjective expected utility model (e.g., Edwards, 1962) *at a specific moment*.

When the weights represent the importance of an attribute or dimension, then the valence of each act is equivalent to a weighted sum of multiattribute values (see von Winterfeldt and Edwards, 1986; Elman, chapter 8) *at a specific moment*. Diederich (in press) has applied DFT to multiattribute, multiple alternative-choice situations.

The effect of physical distance on the weights is referred to as the goal gradient hypothesis (cf., Hull, 1938; Lewin, 1935; Miller, 1944). Sensations associated with the reward or punishment become much more salient as one approaches the goal. For example, a hungry dieter may be able to see and smell food better as he or she approaches it. The fear of a soldier approaching battle rises as the thunder from the guns grows louder.

One final point is that the weights depend on the sign of a consequence. This assumption is based on the principle that avoidance gradients are steeper than approach gradients (Lewin, 1935; Miller, 1944). For example, positive and negative consequences associated with equal delays and probabilities receive different weights.

The motivational value vector, $\mathbf{M}(t)$, represents the decision-maker's overall affective reaction to each of m possible consequences. Here we assume that all desires, feelings, and emotional reactions to consequences can be temporarily mapped onto a single common underlying scale similar to Wundt's hedonic continuum (see Cofer and Appley, 1964). The explicit purpose of this continuum is to compare consequences and make tradeoffs within a single biological system in a manner similar to the way that a monetary continuum is used in economic systems for trading between individuals. This is not to say that feelings and emotions are one-dimensional. On the contrary, motivational value is only a summary of these many dimensions temporarily constructed for the purpose of guiding action.

This is where the internal needs, demands, or motivational states of the decision-maker enter the decision process. Motivational value is derived from the product of two factors: (a) internal demands or drives, and (b) the estimated potential for a consequence to supply or satisfy these demands. The dynamic nature of motivational value now becomes apparent. First, demands often grow over time producing an increase in motivational value. Second, actions yield consequences that satisfy these demands, and reduce motivational value (cf. Atkinson and Birch, 1970). Finally, experience with consequences modifies one's estimate of the potential satisfaction.

The motivational values are positively or negatively signed. Positive values attract a person toward a goal, negative values repel a person away from a goal, and zero represents a neutral point. The sign of the values has an important influence on the dynamic process. For example, avoidance processes induced by two negative acts produce more vacillation than approach processes induced by two positive acts, even when the difference in value between two negatives equals that for two positives. The neutral or reference point can be influenced by context or "framing" (see Helson, 1959; Parducci, 1974; Tversky and Kahneman, 1981).

This concludes our overview of the general theoretical structure. For any given decision task, a specific mathematical model can be constructed from this general framework, and quantitative empirical tests can be derived. In the next section, we outline two different specific models—one for choice tasks and another for selling price tasks. Note that only the motor system changes across these two decision tasks, and the valence and decision systems are assumed to remain invariant across these two measures of preference.

4.3 RESPONSE MODELS FOR CHOICE AND SELLING PRICE TASKS

Binary Choice Response Model

Suppose the decision-maker is asked to choose between two actions by pushing either a left or right response key on a computer. Figure 4.2 gives an outline of the basic ideas of the choice model for this situation. The horizontal axis indicates the deliberation time, and the vertical axis represents the difference in preference states between the right and left actions (positive differences produce a tendency to move toward the right key; negative differences produce a tendency to move toward the left key). The polygonal line is a sample path of the difference in preference states during deliberation, and note that it wanders up and down as the decision-maker considers the various consequences of each action. The flat lines located at the top and bottom of the figure are called the inhibitory thresholds. No movement is emitted until the difference in preference states exceeds or overcomes this inhibitory threshold magnitude. If the upper threshold is exceeded before the lower threshold, then the right key is pushed. The vertical line on the right-hand side of the figure indicates the time required to exceed the threshold and make the decision.

Realistically, the inhibitory threshold would start at some large magnitude at the beginning of deliberation, and gradually weaken or decay toward zero as the deliberation process continued. However, for simplicity, the inhibitory threshold was fixed to a constant value for the predictions computed in the applications described later.

In sum, the first act to exceed the threshold wins the race and determines the choice. The probability of choosing each action is given by the probability that an action will win the race, and the time required to make the de-
[illegible] exceed the threshold. (See

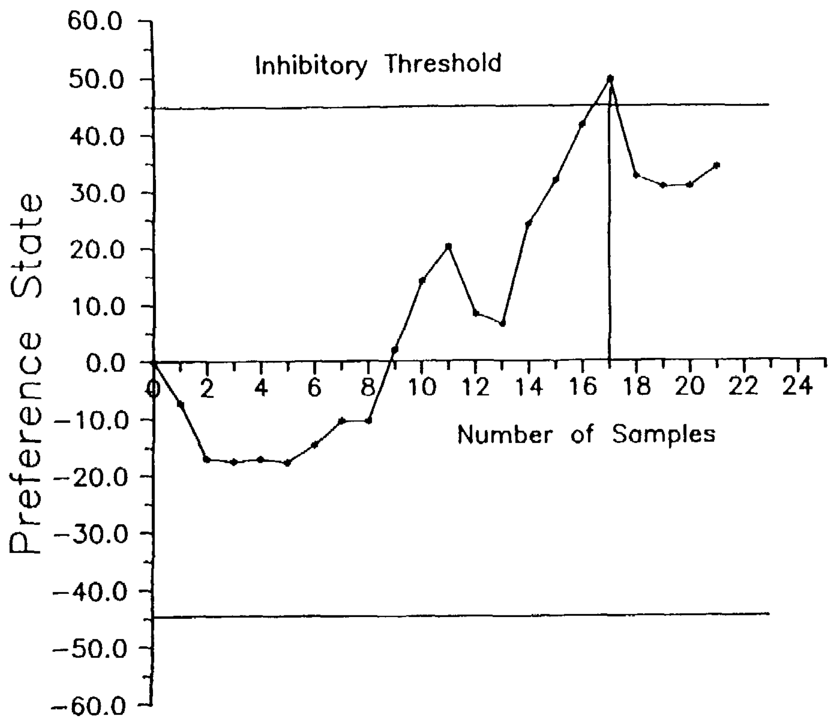


Figure 4.2 Choice model for DFT. The horizontal axis represents time or the number of consequences sampled during deliberation. The vertical axis represents the difference in preference states between two actions. Positive values represent a preference for taking the action located on the right, and negative values represent a preference for taking the action on the left. The horizontal lines parallel to the horizontal time axis represent threshold bounds. The action on the right is taken as soon as the difference in preference exceeds the upper bound, and the action on the left is taken as soon as the difference in preference exceeds the lower bound.

Busemeyer and Townsend, 1992, for the derivation of the mathematical formulas used to compute the choice probabilities and mean response times.)

Indifference Response Model

In some choice tasks, the decision-maker is permitted to express indifference. Consider the case where three options are available: (1) press the left key for one action, (2) press the right key for a second action, or (3) press the middle key for indifference. The binary choice model described above is extended to allow for indifference responses as follows. Each time the difference in preference states crosses zero (the neutral point), there is a probability that the decision-maker will stop and push the indifference response key. The probability of stopping and pushing the indifference key in the neutral state is called the exit probability. (See Busemeyer and Townsend, 1992, for the derivation of the mathematical formulas used to compute the choice probabilities for the indifference response.)

In general, the exit probability would be zero at the beginning of deliberation and gradually increase during deliberation. However, for simplicity, the exit probability was fixed to a constant value for the predictions computed in the applications described later.

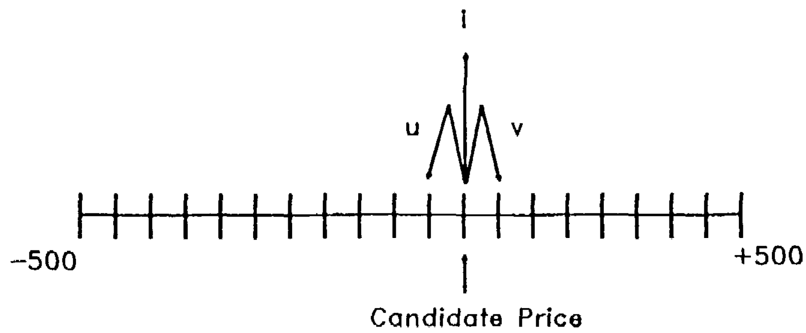
Dynamic Matching Model

Suppose a decision-maker owns a risky venture which could yield a win of \$500 or a loss of \$500 with equal probability. Now the owner considers selling this investment, and the minimum selling price needs to be determined. The minimum selling price is the price at which the decision-maker is indifferent between keeping the risky investment or taking the cash value. This is closely related to finding the cash equivalent of a investment. Below we present an account of how the minimum selling price or cash equivalent is estimated by a dynamical matching process.

According to DFT, the decision-maker proceeds through a series of hypothetical choices between the investment and candidate prices until an indifference response is elicited with a candidate price. The matching process starts by considering a hypothetical choice between the investment and an initial selling price (e.g., the midpoint between the minimum and maximum possible price). If the choice produces a response favoring the selling price, then this initial price is too high, and the price is decreased by a fixed amount, δ. If the choice produces a response favoring the investment, then this initial price is too low, and the price is increased by a fixed amount, δ. In both of the above cases, the matching process is repeated using the newly adjusted price. This matching process continues until the choice between the investment and a candidate price elicits an indifference response, at which point the price currently being considered is selected and reported.

The matching process is illustrated in figure 4.3. The horizontal axis represents candidate prices, with the minimum and maximum points fixed by the minimum and maximum amounts that can be obtained from the investment. The point indicated by the arrow in the figure represents a candidate price currently being considered for the investment. There is a probability u of making a step down, which is determined by the probability of choosing the current price over the investment. There is another probability v of making a step up, which is determined by the probability of choosing the investment over the current price. Finally, there is a probability i of choosing the indifference response, which would terminate the matching process at the current price. (See Busemeyer and Townsend, 1992, for the derivation of the mathematical formulas used to compute the distribution of selling prices for an investment.)

This matching process is not restricted to selling prices or cash equivalents. For example, it can also be used to find probability equivalents. In the latter case, the decision-maker is asked to find a probability value that makes him or her indifferent between a gamble and a fixed cash value. In this case, the matching process is applied to the probability scale, and the decision-maker



u = Pr[step down] = Pr[prefer candidate price]
v = Pr[step up] = Pr[prefer gamble]
i = Pr[report as final price] = Pr[being indifferent]

Figure 4.3 Dynamic matching model. A candidate price is compared to the gamble that is being evaluated. A choice favoring the price is made with probability u, in which case the price is reduced by an increment. A choice favoring the gamble is made with probability v, in which case the price is increased by an increment. An indifference response is made with probability i, in which case the matching process terminates, and the current price is selected.

performs a sequence of tests of probability values until an indifference response is elicited.

In sum, minimum selling prices, cash equivalents, and probability equivalents are determined by the binary choice and indifference response models discussed previously. Thus, the same parameter values used to compute the predictions for choice probabilities in binary choice tasks can also be used to compute the distribution of prices selected in a selling price task. In the next section, we show how the theory provides a simple explanation for what were previously considered "paradoxical" findings from the view of more traditional static-deterministic derivatives of expected utility theory.

4.4 INCONSISTENCIES AMONG PREFERENCE MEASURES

Static-deterministic decision theories (such as expected utility theory and its variants) generally assume that decision-makers can precisely and reliably determine their minimum selling price. In other words, static-deterministic theories are based on the solvability axiom which states that decision-makers can solve for the unique price such that they are indifferent between keeping an investment or taking the cash value. In fact, empirical research indicates that decision-makers are not very reliable in their estimates of minimum selling prices or cash equivalents. Schoemaker and Hershey (1993) reported a test-retest correlation as low as .50 from management students who were asked to give cash equivalents for simple gambles 1 week apart in time. Although most decision theorists readily acknowledge that selling prices are unreliable, the theoretical implications of this fact have not been thoroughly explored. Below, we show that two different "paradoxical" findings from decision research can be explained as an emergent property of the fundamental stochastic and dynamical nature of preference.

Inconsistencies Between Choice and Selling Price

The two most popular methods for measuring an individual's preference between two actions is the *choice method* and the *minimum selling price method.* According to traditional deterministic-static decision theories, if L and R are two actions, then each action can be assigned a utility, u(L) and u(R) respectively, such that if u(R) > u(L), then action R should be chosen over action L, and the selling price for R should be greater than the selling price for L. In other words, the preference order measured by the choice method should be consistent with the preference order measured by the selling price method.

One "paradoxical" finding in the decision-making literature is that under certain well-known conditions, the preference ordering measured by choice systematically disagrees with the preference ordering measured by selling price (Lichtenstein and Slovic, 1971; Lindman, 1971; see Slovic and Lichtenstein, 1983, for a review). In particular, suppose the decision-maker is asked to consider two gambles called the P-bet and the D-bet. Both gambles are approximately equal in expected value, but the P-bet has a high probability of winning a small amount (e.g., .99 probability of winning $4 or else nothing), and the D-bet has a low probability of winning a large amount (e.g., .33 probability of winning $12 or else nothing). The usual finding is that the P-bet is chosen more frequently over the D-Bet, but the selling price for the D-bet is more frequently larger than the selling price for the P-bet. This finding has even been replicated at a Las Vegas gambling casino using casino players and real money (Lichtenstein and Slovic, 1973)!

Previous theoretical explanations for this type of preference reversal finding have been based on the idea that changing the way preference is measured from choice to selling price changes the parameters that enter the calculation of the utility of each gamble. For example, Tversky, Sattath, and Slovic (1988) hypothesized that individuals assign separate weights to the probability dimension and the value dimension for gambles of the form "win X with probability P." Furthermore, they hypothesize that these weights change depending on whether the individual is asked to make a choice or select a selling price. To account for the preference reversal finding, they assume that more weight is given to probability in the choice task, but more weight is given to the payoff value in the selling price task. Below, we provide an alternative explanation which does not require changes in parameters to account for the inconsistencies between choice and selling price.

It turns out that this "paradoxical" inconsistency in preference ordering between choice and selling prices is an emergent property of the dynamic-stochastic choice and selling price models described above. Figure 4.4 illustrates the predictions computed from the mathematical formulas for the

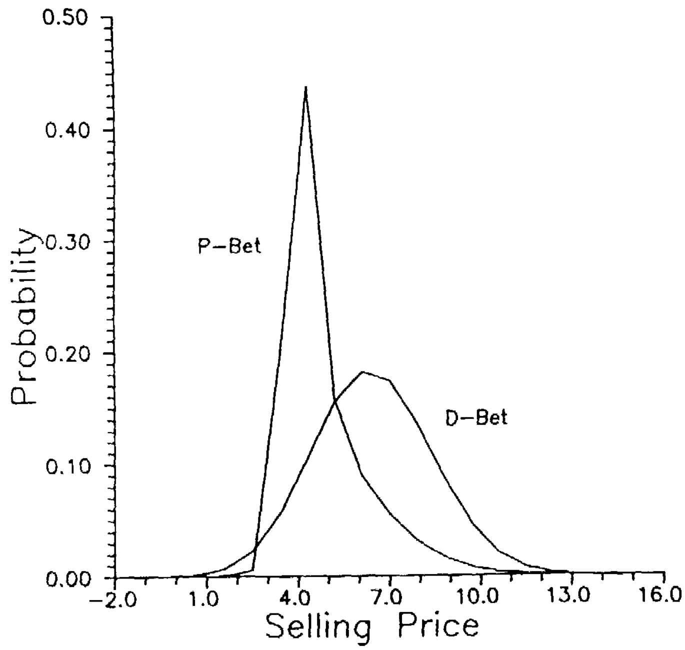


Figure 4.4 Distribution of selling prices predicted by DFT for the P-bet and the D-bet. The horizontal axis represents the various possible prices. The vertical axis represents the relative frequency that a price is selected. Predictions were generated using the exact same parameter values that produced a probability of choosing the P-bet over the D-bet equal to .56.

choice and selling price models (see Busemeyer and Goldstein, 1992, for more details).

First, the parameters of the choice model were selected so that the predictions from the choice model accurately reproduced the observed choice relative frequencies. The figure illustrates an example where the probability of choosing the P-bet over the D-bet was predicted by the choice model to be .56, which is approximately the same as the empirically observed proportions for this particular example choice problem.

Second (and this is the crucial point), these *same exact parameter values* were used to calculate predictions from the selling price model to produce the distributions of minimum selling prices shown in the figure. Note that the predicted distribution of selling prices for the P-bet lies below that for the D-bet, consistent with the observed results. Furthermore, note that the variance of the D-bet distribution is predicted to be larger than that for the P-bet, which is also consistent with known results (e.g., Bostic, Herrnstein, and Luce, 1990). Thus, a systematic reversal in the preference ordering was predicted using the same parameter values for the valence and decision systems and simply changing the motor system.

The precise reason that DFT produces this reversal in preference is an emergent property of the interactions of the dynamic and stochastic components of the model. However, a heuristic explanation might help give the reader some intuition about this mechanism. According to the model, the initial candidate selling price starts near the middle of the price scale. Because of the way the gambles are constructed, the middle of the scale produces an overestimate of the price for each gamble. According to the model, it is easy to discriminate the preference difference between the cash value and the worth of the P-bet because of the low variance of the P-bet. This high level of discriminability causes the initial price to quickly adjust down toward the true indifference point. Also according to the model, it is difficult to discriminate the preference difference between the cash value and the worth of the D-bet because of the high variance of the D-bet. This low level of discriminability causes the initial price to slowly adjust down toward the true indifference point, and the process wanders around and stops far short of a complete adjustment needed to reach the true indifference point.

In sum, DFT provides a simple explanation of the inconsistencies between two measures of preference—choice and selling price—in a coherent manner by using the same model parameter values for both tasks. A stronger test of the theory is provided by using these same parameter values once again to account for an inconsistency in preference found with another two measures of preference, described next.

Inconsistencies Between Certainty and Probability Equivalents

Two methods are commonly used by decision analysts to measure the utility in risky investments: one is called the certainty equivalence method and the other is called the probability equivalence method. Hershey and Schoemaker (1985) proposed a two-stage design for testing the consistency of these two ways of measuring utility. In both stages, a measurement of the utility of a gamble of the form "win \$500 with probability P" is obtained. In the first stage, this utility is measured by the certainty equivalence method, and in the second stage it is measured by the probability equivalence method.

In the first stage, the probability of winning is set to $P = .50$, and the decision-maker is asked to find the cash value X that makes him or her indifferent between the cash value (X) and the gamble (win \$200 with probability .50). According to most static-deterministic theories, this problem is solved by finding the value of X such that

$$u(X) = w(.50)u(200),$$

where $w(.50)$ is the decision weight assigned to the probability of winning, $u(200)$ is the utility of \$200, and $u(X)$ is the utility of the cash value X. For example, suppose the decision-maker is indifferent between the cash value of $X = \$75$ and the gamble "win \$200 with probability .50."

In the second stage, utility is measured by a probability equivalence method. This is accomplished by asking the decision-maker to find the probability P such that he or she is indifferent between the cash value X and the gamble "win \$200 with probability P" where X is the same cash value obtained from the first-stage task. For example, if $X = \$75$ was selected in the first stage, then the decision-maker is asked to find P such that he or she is indifferent between the cash value of $75 and the gamble "win $200 with probability P." According to most static-deterministic theories, this problem is solved by finding the probability P such that

$$u(X) = w(P)u(200).$$

Obviously, to be consistent across both stages, the decision-maker should choose $P = .50$, the original probability value used to determine X in the first stage. In other words, according to static-deterministic utility theories, the P value selected in the second stage should always be set to $P = .5$, independent of the value of X chosen in the first stage.

According to static-deterministic theories, the variance of the P value selected in the second stage should be zero. In fact, Hershey and Schoemaker (1985) found a considerable amount of variability in the second-stage P value. More important, the variation in the P value was systematically related to the payoff value X selected in the first stage. The observed correlation between the first-stage value of X and the second-stage value of P was $r = .67$. This runs counter to static-deterministic theories which predicted zero correlation between the two stages. These results were later replicated by Schoemaker and Hershey (1992) and Johnson and Shkade (1989).

Table 4.2A provides a summary of the results of the experiments by Hershey and Schoemaker (1985) and Schoemaker and Hershey (1992). This table was constructed as follows. First, the monetary scale was rescaled to match the probability scale (i.e, we replaced X with $X/200$). Then both scales were partitioned into three response categories: $[0, .45)$, $[.45, .55]$, and $(.55, 1.0]$. Each cell of the table indicates the proportion of 300 subjects that made responses within each of the nine categories formed by crossing the three first-stage categories with the three second-stage categories.[1] For example, the middle row indicates the proportion of subjects selecting second-stage values within the interval $[.45, .55]$. According to static-deterministic theories, all of the responses should fall into this interval. Instead, the table shows a strong posi-

Table 4.2 Observed (A) and predicted (B) relative frequencies of probability and certainty equivalents from Hershey and Shoemaker experiments

Second-stage value	First-stage value		
	0.0–.55	.45–.55	.56–1.0
A. Observed relative frequencies for gain conditions			
.56–1.0	.07	.04	.25
.45–.55	.12	.10	.12
0.0–.44	.20	.04	.06
B. Predicted relative frequencies from DFT using the same parameter values as in figure 4.3			
.56–1.0	.08	.07	.27
.45–.55	.08	.06	.06
0.0–.44	.28	.04	.06

Data from Hershey and Shoemaker (1985) and Schoemaker and Hershey (1992).

tive correlation between the first- and second-stage selections. Note that the largest frequencies occur in the lower-left and upper-right corner cells.

The predictions computed from DFT are shown in table 4.2B. It is important to note that we used the *exact same parameter values* in table 4.2B that were used in figure 4.4. Also note that the model accurately reproduces the positive correlation between first- and second-stage results. Thus, the systemic discrepancies between certainty and probability equivalence methods for measuring utility can be explained without postulating changes in utilities across tasks. Instead, the discrepancies can be explained as the result of the dynamical-stochastic processes required to perform these two tasks.

4.5 CONCLUDING COMMENTS

For the past 50 years, the field of decision-making has been dominated by static-deterministic theories. While these theories have provided a useful first approximation to human decision-making behavior, they fail to describe two very basic facts about human decision-making behavior—the variability and the temporal evolution of preferences. We think it is time to consider a better second-order approximation to human decision-making that captures these two basic properties of human preference. In this chapter, we presented an alternative approach called decision field theory (DFT) which provides a dynamical-stochastic description of decision-making. Furthermore, we showed that what often appears to be "paradoxical" decision behavior from the point of view of static-deterministic theories can be understood as emergent properties of the dynamical-stochastic process that individuals use to perform decision tasks.

NOTE

1. The data in table 4.2A come from experiments by Hershey and Schoemaker (1985) and Schoemaker and Hershey (1992). Each experiment had four conditions: Two conditions employed gambles which produced gains of the form "win $200 with probability *P*," and the other two conditions employed gambles which produced losses of the form "lose $200 with probability *P*." Table 4.2 only includes the conditions which employed gains because we used parameter values from previous research on preference reversal that were restricted to gains. The two gain conditions differed according to the task order: the certainty equivalent task in stage 1 followed by the probability equivalent task in stage 2; in the other condition the opposite task order was used. For both task orders, static-deterministic theories predict that all of the subjects should fall into the middle row of table 4.2. We pooled the data across both task orders, and we also pooled across both experiments to produce the results shown in table 4.2.

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Guide to Further Reading

A recent summary of the traditional static-deterministic approach to decision-making can be found in Kleindorfer, Kunreuther, and Schoemaker (1993). For a review of research on preference reversals for choice and selling price measures of preference, see Slovic and Lichtenstein (1983). A more thorough presentation of the empirical support for DFT can be found in Busemeyer and Townsend (1993). All of the mathematical equations used to calculate the predictions from DFT can be found in Busemeyer and Townsend (1992). An extension of DFT to multiattribute decision problems has been developed by Diederich (in press). An alternative dynamical model of decision-making is described in Grossberg and Gutowski (1987).

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