Alphabetic confusion: A test of models for individuals*†

J. T. TOWNSEND
Purdue University, West Lafayette, Indiana 47907

An earlier article reported extensive analyses of confusion data compiled from group averages (Townsend, 1971). The present study provided for essentially the same analyses with different long-term data obtained with two individuals, the primary intent being to examine the ability of recognition and scaling models to explain data at the individual level (which the recognition models purport to describe) and to compare the confusion characteristics of the English uppercase alphabet between the two Ss and between the individual Ss and the group-averaged data. The choice and overlap models were superior to the all-or-none model in predicting the empirical confusion matrices and tended to explain the data structure in a similar manner. Multidimensional scaling analysis again supported a Euclidean metric and suggested four or five underlying stimulus dimensions. However, as before, there were no overriding intuitively appealing psychological dimensions corresponding to these, and possible reasons are discussed. The choice and overlap models appeared to fit as well or better at the individual level than at the group level and the all-or-none model to fit worse. In the present study, probability correct was fit even better by the all-or-none model than in the group study and replicated the result of being better here than the overlap and choice models. Individuals and the group were consistent in their sensory confusions as represented by similarity parameters in the choice and overlap models but differed in their response biases. A simple measure of physical similarity explained 50% of the variance of the similarity structure in the confusion data.

An earlier article (Townsend, 1971) presented a group-average confusion matrix for the complete uppercase English alphabet obtained in a tachistoscopic recognition experiment. Three models of recognition were developed, tested, and compared in their predictions for the group data. The three models were: (1) all-or-none-activation model, (2) overlap activation model, and (3) choice model, the first two being derived from a general class of finite-state recognition models and the third being as presented in Luce (1963). The second and third contain structure for representing sensory confusions, but the first assumes only perfect perception or guessing at random. The appendix gives the theoretical formulae for the three models. In addition, the similarity parameter in the choice model was used as input to the Kruskal multidimensional scaling program in order to attempt to establish pertinent dimensions of visual confusion. Finally, correlations were obtained among parameters for the three models and between the two experimental conditions (with and without postceding visual noise), and between similarity measures, scaled distances, and a physical measure of alphabetic similarity. It was of interest to replicate the experiment with long-term individual data for the following reasons: (1) to provide parametric complete confusion data for the alphabet at the individual level, there being too few trials for this purpose in the group experiment, (2) to test the models, which presume to describe behavior for the individual S, with respect to individuals, and (3) to compare the scaling results for the group averages with those of individuals.

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METHOD

Apparatus

All apparatus materials, and luminance and size characteristics, were as reported for the group data in Condition I (without postceding visual noise) and will be referred to but briefly here.

The presentation device was a Gerbrands Model T-2B-1 tachistoscope employed to replicate Condition I with measured luminance of 5.5 fL for the prestimulus, stimulus, and poststimulus fields. The capital letters of the alphabet were presented one at a time over a fixation point from a shuffled deck containing five alphabets, each letter subtending about ½ deg at S's eye, and typed with an IBM Executive Directrix typewriter. An intratrial sequence was (1) blank prestimulus field with fixation point, (2) stimulus field with letter, and (3) poststimulus field, identical to prestimulus field.

Procedure and Subjects

The stimulus presentations were S-paced following an alert buzzer, after which S gave a response followed by feedback as to the correct response. The Ss were two University of Hawaii coeds with good vision, who were paid for their participation in the experiment. The Ss were practiced and calibrated for 4 days before beginning the experiment proper. Their data appeared stable by the end of the practice period. Following the training period, they were each run for 30 days at 130 trials per day, resulting in 150 presentations for each letter for each S.

The computer programs for the recognition models were identical to those used in the previous group study. The multidimensional scaling program used was Torscal, written by Young (1968); that employed with the earlier group averages was Kruskal's (1964).

RESULTS AND DISCUSSION

Tables 1A, 1B show the empirical confusion matrices for the two Ss (MJ and VF).[1] The general pattern of confusions appears to be similar to that previously found with the averaged data. Some representative predictions of probability correct and confusion probabilities for the letter "P" as stimulus are shown in Table 2. The empirical entries correspond to intuition about which letters bear physical similarity to "P," and match the earlier group results. For example, higher confusion values are revealed from "P" to "B," "F," and "R" than to other letters for the individuals, as for the group. The most apparent deviation from the group results here were that these exhibited some confusion of the letters "H" and "O" with "P," but the individual data did not. The reason for this difference is not clear at present. The theoretical values illustrate the superior ability of the choice and overlap models to predict confusion and, for MJ, the superiority of the all-or-none model on probability correct. There are some interesting asymmetries in the confusion matrix that may point to sensory-bias interactions. For example, both of the present Ss showed more confusions of "E" to "F" than from "F" to "E." This kind of asymmetry might be expected if on some trials the feature represented by the lower horizontal component of the "E" were missing and this sensory state were then more heavily associated with those letters lacking that feature. Now, this interpretation is consistent with the general overlap type of model. But to produce this

Table 1A
Empirical Confusion Matrix (MJ)

STIMULUS	RESPONSE																									
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	.76	.00	.00	.00	.01	.02	.00	.00	.01	.03	.03	.00	.02	.01	.01	.00	.01	.00	.01	.00	.00	.01	.00	.02	.01	.04
B	.02	.48	.01	.04	.05	.00	.01	.05	.01	.04	.01	.02	.02	.02	.02	.03	.00	.08	.02	.01	.00	.01	.01	.00	.01	.01
C	.01	.02	.37	.02	.12	.06	.10	.00	.01	.00	.07	.06	.00	.00	.02	.02	.04	.02	.00	.01	.00	.01	.01	.01	.01	.01
D	.00	.08	.01	.51	.01	.01	.01	.03	.01	.02	.00	.00	.01	.02	.08	.03	.06	.01	.03	.00	.04	.01	.01	.00	.00	.01
E	.02	.02	.05	.00	.41	.15	.02	.00	.07	.01	.02	.08	.00	.01	.01	.02	.01	.01	02	.04	.00	.01	.01	.01	.00	.00
F	.02	.01	.01	.00	.08	.43	.00	.01	.13	.06	.03	.02	.00	.01	.01	.07	.00	.01	.01	.04	.00	.02	.01	.01	.01	.00
G	.01	.08	.12	.02	.07	.01	.34	.01	.02	.01	.01	.01	.01	.02	.10	.03	.02	.01	.03	.00	.02	.01	.02	.01	.00	.01
H	.01	.02	.00	.00	.01	.02	.00	.49	.06	.05	.02	.01	.06	.11	.01	.01	.00	.03	.00	.00	.02	.01	.03	.00	.00	.00
I	.00	.00	.00	.00	.02	.04	.00	.01	.65	.08	.02	.09	.00	.00	.00	.00	.00	.00	.00	.06	.00	.00	.00	.01	.01	.01
J	.01	.00	.00	.01	.00	.02	.00	.00	.31	.48	.01	.01	.00	.00	.02	.00	.00	.01	.01	.04	.00	.02	.01	.00	.01	.04
K	.03	.01	.02	.00	.02	.06	.01	.02	.06	.01	.50	.01	.01	.02	.00	.00	.00	.05	.04	.02	.01	.01	.00	.05	.02	.02
L	.01	.00	.03	.01	.14	.03	.01	.01	.24	.01	.01	.44	.00	.00	.00	.01	.00	.00	.01	.02	.01	.01	.00	.00	.00	.01
M	.06	.01	.00	.01	.00	.00	.00	.04	.02	.01	.02	.00	.69	.04	.01	.01	.01	.01	.01	.00	.00	.01	.02	.01	.00	.00
N	.06	.01	.01	.00	.01	.00	.00	.02	.01	.00	.10	.00	.07	.52	.01	.00	.00	.03	.01	.00	.02	.03	.05	.03	.01	.01
O	.01	.01	.03	.10	.02	.00	.11	.01	.00	.01	.00	.00	.00	.00	.49	.01	.14	.01	.00	.00	.04	.00	.01	.00	.00	.00
P	.01	.04	.01	.01	.01	.14	.00	.01	.04	.01	.02	.00	.01	.01	.01	.53	.01	.07	.03	.01	.00	.00	.00	.00	.00	.02
Q	.01	.01	.05	.04	.01	.00	.09	.01	.00	.01	.01	.00	.00	.00	.28	.01	.40	.02	.02	.00	.02	.01	.01	.00	.00	.00
R	.04	.08	.01	.01	.02	.04	.01	.01	.02	.01	.07	.01	.02	.04	.00	.06	.02	.42	.04	.01	.00	.02	.01	.02	.00	.02
S	.02	.03	.01	.01	.01	.01	.01	.01	.02	.01	.02	.01	.00	.01	.00	.01	.01	.02	.67	.00	.00	.02	.01	.04	.01	.04
T	.00	.01	.00	.00	.01	.04	.01	.00	.34	.07	.01	.04	.00	.00	.00	.00	.00	.00	.00	.41	.01	.01	.00	.00	.01	.02
U	.01	.04	.01	.05	.01	.01	.00	.13	.01	.06	.01	.04	.01	.05	.04	.01	.01	.00	.01	.00	.44	.04	.02	.00	.00	.00
V	.01	.00	.00	.00	.00	.01	.01	.00	.01	.01	.02	.01	.01	.02	.00	.00	.01	.00	.01	.00	.03	.69	.01	.01	.10	.05
W	.04	.00	.01	.01	.00	.01	.01	.03	.02	.01	.03	.01	.06	.08	.01	.01	.00	.01	.02	.00	.01	.05	.56	.01	.01	.00
X	.03	.01	.01	.00	.01	.00	.00	.00	.04	.02	.13	.00	.01	.01	.00	.01	.00	.01	.02	.01	.00	.02	.02	.58	.04	.04
Y	.01	.00	.02	.00	.00	.01	.00	.01	.04	.01	.03	.01	.00	.00	.00	.00	.00	.00	.02	.02	.01	.11	.01	.07	.60	.02
Z	.01	.00	.02	.00	.00	.02	.00	.01	.04	.01	.01	.01	.00	.01	.00	.00	.00	.00	.01	.02	.01	.01	.01	.03	.02	.78

interaction, it would be necessary to employ a more general bias variable than was used in the present overlap model. The probability of saying F given a (overlap model) confusion between E and F is $g_F/(g_E + g_F)$ where g_F, g_E are the individual biases or response strengths associated with the letters F, E. But, to produce the present interpretation, it would be necessary to allow a bias $g_{E,F}$ which takes account of the special relation between the two letters. This, of course, can substantially complicate the model, as shown in the earlier paper.

As was the case for the group study, the overlap and choice models are able to predict confusion much better than the all-or-none model but the latter is adequate for prediction of probability correct. Table 3 shows this and also indicates that the comparison of the sum of squared deviations of theoretical from empirical values with that expected from a model predicting uniform confusion matrix values (EP in Table 3) is similar to that obtained earlier. (For a more detailed explication of this and other analyses, the reader is referred to the earlier paper.) Another point to note is that the all-or-none model is less effective with the individual than with the group data; the measure of fit for the group was .49 for the all-or-none model. Also, the disproportionate contribution to the total sum of squared deviations from the off-diagonal entries for the all-or-none model is even greater with the individual data. This suggests that averaging over Ss may obscure some differences in confusion structure and thereby benefit the all-or-none model. Another deviation from the group results is that the overlap model appears closer to

Table 1B
Empirical Confusion Matrix (VF)

RESPONSE

STIMULUS	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	.72	.01	.01	.01	.01	.00	.00	.01	.01	.01	.04	.00	.00	.03	.00	.01	.00	.03	.03	.00	.01	.00	.00	.06	.00	.01
B	.03	.46	.01	.03	.04	.03	.02	.03	.01	.01	.01	.01	.01	.04	.02	.03	.01	.14	.04	.00	.01	.01	.01	.00	.01	.00
C	.01	.01	.65	.00	.09	.03	.05	.01	.01	.00	.01	.04	.00	.00	.01	.01	.01	.01	.01	.00	.00	.00	.00	.02	.00	.01
D	.00	.06	.01	.56	.01	.01	.03	.02	.00	.01	.00	.01	.01	.01	.07	.08	.01	.02	.02	.00	.03	.01	.00	.01	.01	.00
E	.02	.00	.06	.00	.26	.19	.01	.00	.06	.01	.06	.10	.00	.01	.00	.01	.00	.02	.01	.06	.00	.01	.00	.02	.02	.06
F	.00	.00	.03	.00	.05	.42	.00	.00	.13	.09	.02	.02	.00	.00	.00	.03	.00	.02	.02	.10	.00	.00	.00	.02	.02	.01
G	.03	.09	.17	.03	.03	.01	.43	.01	.00	.01	.01	.01	.00	.01	.04	.01	.04	.04	.02	.00	.02	.00	.00	.00	.00	.00
H	.03	.02	.00	.00	.00	.01	.00	.58	.00	.00	.00	.01	.02	.17	.01	.00	.01	.01	.00	.00	.10	.01	.03	.01	.00	.00
I	.01	.00	.00	.00	.01	.01	.00	.00	.44	.23	.01	.06	.00	.00	.00	.00	.00	.01	.02	.15	.00	.00	.00	.02	.02	.00
J	.02	.00	.00	.00	.00	.02	.00	.01	.10	.66	.01	.01	.00	.00	.00	.00	.00	.00	.01	.04	.04	.00	.00	.04	.01	.03
K	.05	.01	.04	.00	.02	.05	.00	.02	.02	.00	.43	.01	.01	.03	.01	.01	.00	.04	.02	.01	.01	.01	.00	.15	.04	.01
L	.01	.00	.04	.00	.03	.03	.00	.00	.12	.03	.03	.59	.00	.01	.00	.01	.00	.02	.01	.02	.00	.00	.00	.04	.01	.01
M	.02	.00	.01	.01	.00	.00	.00	.01	.01	.00	.00	.00	.77	.05	.00	.01	.01	.02	.01	.00	.01	.00	.05	.01	.00	.00
N	.05	.01	.01	.01	.00	.00	.00	.02	.01	.01	.02	.02	.04	.68	.01	.00	.01	.02	.01	.00	.00	.01	.05	.01	.01	.00
O	.01	.02	.03	.19	.00	.00	.09	.00	.00	.00	.00	.00	.00	.01	.33	.00	.25	.02	.01	.00	.03	.00	.01	.00	.00	.00
P	.01	.04	.01	.01	.02	.08	.02	.01	.00	.00	.00	.01	.01	.00	.01	.69	.00	.05	.02	.00	.01	.00	.00	.01	.00	.01
Q	.01	.02	.01	.14	.00	.00	.10	.01	.00	.00	.00	.00	.00	.00	.19	.00	.47	.01	.00	.00	.03	.00	.01	.00	.00	.00
R	.06	.06	.03	.01	.01	.01	.03	.04	.01	.00	.04	.00	.03	.03	.01	.12	.01	.42	.03	.00	.02	.00	.01	.01	.01	.00
S	.01	.03	.02	.01	.00	.03	.03	.01	.01	.00	.01	.00	.00	.01	.01	.00	.00	.03	.74	.01	.00	.00	.00	.02	.01	.01
T	.01	.00	.01	.00	.01	.08	.00	.00	.10	.13	.02	.02	.00	.00	.00	.00	.00	.00	.01	.49	.00	.00	.00	.01	.08	.03
U	.00	.01	.02	.02	.00	.00	.01	.04	.00	.01	.00	.02	.02	.03	.04	.00	.01	.02	.00	.01	.70	.01	.02	.00	.00	.00
V	.01	.01	.00	.00	.01	.01	.00	.00	.01	.01	.01	.00	.00	.02	.01	.01	.00	.01	.01	.00	.04	.64	.02	.04	.13	.01
W	.02	.02	.00	.01	.01	.00	.03	.03	.00	.00	.02	.00	.06	.02	.02	.00	.02	.02	.01	.00	.02	.01	.65	.01	.00	.00
X	.01	.01	.01	.01	.00	.00	.00	.00	.00	.00	.15	.00	.01	.02	.01	.00	.01	.01	.04	.00	.00	.01	.01	.68	.03	.01
Y	.01	.01	.01	.00	.01	.01	.00	.00	.01	.02	.02	.00	.00	.01	.00	.00	.00	.01	.02	.02	.02	.06	.01	.11	.60	.03
Z	.01	.00	.05	.00	.00	.03	.00	.00	.01	.03	.01	.00	.00	.01	.00	.01	.00	.00	.02	.01	.00	.00	.01	.08	.01	.73

the choice model in its ability to predict probability correct in the present data. Finally, we should note that taking sums of squared deviations gives a relative advantage to the all-or-none model since an absolute deviation for an off-diagonal value equal in proportion to that of a diagonal value will not contribute equally to the sum of squared deviations.

In the previous study, the similarity parameters of the choice model were input to Kruskal's (1964a, b) multidimensional scaling program in order to attempt some conclusions as to the perceptual space underlying tachistoscopic alphabetic recognition. The fit as measured by "stress" showed the Euclidean metric to be superior to the city-block metric and although there are as yet no accepted statistical tests for ordinal multidimensional scaling programs, Klahr's (1969) Monte Carlo investigations suggested the fits to be acceptable. By the time the present data were ready for analysis, Young's (1968) nonmetric scaling program (intended as an extension and improvement of Kruskal's program) had become available and was used to analyze $-\ln(\eta_{ij})$ where η_{ij} is the similarity parameter in the choice model, the function being a metric suggested by Luce (1963).

Figures 1 and 2 show the index of fit used by Young as a function of number of dimensions in the space for the Euclidean and city-block metrics. The results are similar for the two Ss, with the largest difference, in favor of the Euclidean, emerging around the "elbow." As for the

Table 2
Representative Theoretical Predictions*

RESPONSE

STIMULUS: VF

Stimulus	Model	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
P	EMP	.01	.04	.01	.01	.02	.08	.02	.01	.00	.00	.00	.01	.01	.00	.01	.69	.00	.05	.03	.00	.01	.00	.00	.01	.00	.01
P	AON	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.71	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
P	OLP	.01	.03	.01	.04	.01	.05	.01	.00	.00	.00	.01	.01	.01	.00	.00	.69	.00	.08	.02	.00	.00	.00	.00	.00	.00	.01
P	CHC	.01	.04	.01	.03	.01	.06	.01	.00	.00	.00	.00	.01	.01	.00	.00	.70	.00	.10	.00	.00	.00	.00	.00	.00	.00	.01

STIMULUS: MJ

Stimulus	Model	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
P	EMP	.01	.04	.01	.01	.01	.14	.00	.01	.04	.01	.02	.00	.01	.01	.01	.53	.01	.08	.03	.01	.00	.00	.00	.00	.00	.02
P	AON	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.54	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
P	OLP	.01	.03	.01	.02	.02	.11	.01	.01	.03	.00	.01	.00	.01	.00	.01	.59	.01	.06	.02	.01	.00	.00	.01	.00	.00	.01
P	CHC	.00	.04	.01	.02	.02	.13	.00	.02	.00	.00	.00	.00	.02	.00	.01	.62	.01	.08	.03	.00	.00	.00	.00	.00	.00	.00

*EMP = empirical, AON = all-or-none activation model, OLP = overlap activation model, CHC = choice model.

Table 3
Sum of Squared Deviations of Theoretical from Empirical Points*

S	Model: CHC	AON	OLP	EP
	Entire Confusion Matrix			
MJ	.18	.70	.16	7.46
VF	.22	.60	.24	8.71
	Main Diagonal			
MJ	.09	.004	.09	6.53
VF	.08	.003	.11	7.91

*EP = equiprobable predictions

group results, the elbow appeared to be at about four dimensions. Young suggests that an index of fit in excess of .999 is necessary for a satisfactory solution; this is attained for the present data only at the highest dimensions. It should be noted in this connection that Torscal employs what Kruskal refers to · as the secondary approach to handling similarity "ties," and this results in poorer fits than does the primary approach.

The dimensions revealed by the scaling analysis tended to be as ambiguous as for the earlier study, although roundness and jaggedness were of importance. Apparently the previous finding of Euclidean superiority and dimension ambiguity was not simply due to obfuscation of dimensions by group averaging. Three remarks are perhaps pertinent to this aspect of the analysis: (1) The space which is being scaled is the degraded space produced by brief exposures and hence may deviate from that corresponding to considered judgments of similarity. Against this argument is the appearance of the confusion matrix with highest confusion among intuitively similar letters. (2) Another possibility is that the metrics employed and perhaps orthogonal dimensions are inappropriate to describe the perceptual space in the present type of task. For example, if the perceptual result of each trial is a set of features nonuniquely related to several letters, from which S must make his decision, then the proper metric may be some measure of distinctness of the perceptual representation of features such as that suggested by Restle (1961); $-\log \eta_{ij}$ might be nonmonotonically related to such a metric in the present data. In that case, the appropriate dimensional representation might be a set of multidimensional axes (one for each possible feature) with a 0–1 (present or absent) lattice description. A relatively large number of dimensions might then be required to describe the data. A priori, one might expect a city-block metric to then perform better than the Euclidean since the former would equal the sum corresponding to the union of those features not held in common by two symbols. But this assumes $-\ln \eta_{ij}$ to be monotonically related to the true metric. Also, probabilistic sampling of different features from trial to trial might obscure the lattice structure. On the other hand, if the acquisition of a particular feature is positively or negatively correlated with the acquisition of another, the perceptual result may not be representable by a space with orthogonal dimensions.[2] (3) Finally, these

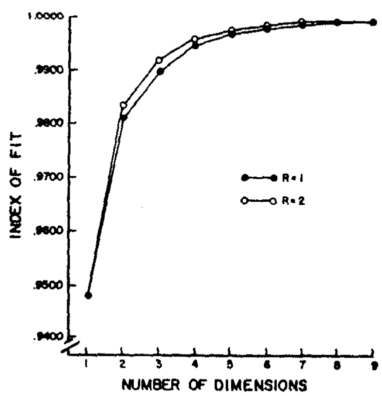

Fig. 1. Spatial fit as a function of number of dimensions (MJ).

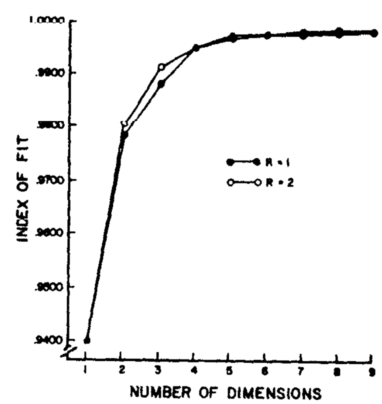

Fig. 2. Spatial fit as a function of number of dimensions (VF).

Table 4
Correlation Coefficients for the Bias Parameters for the Three Models and the Two Ss

Model:		AON		OLP		CHC	
		MJ	VF	MJ	VF	MJ	VF
AON	MJ		−.27	.60		.72	
	VF				.20		.15
OLP	MJ				.10	.75	
	VF						.79
CHC	MJ						.16

Table 5
Correlation Coefficients for Similarity Parameters, Distance Measures, and a Physical Similarity Measure for the Individuals and Individual vs Group Data*

		OLP			CHC			d_c			d_s			
		MJ	VF	GRP	MJ	VF	GRP	MJ	VF	GRP	MJ	VF	GRP	PHOLP
OLP	MJ		.86	.80	.51			−.73			−.62			.54
	VF			.76		.59			−.75			−.69		.56
CHC	MJ					.99	.99	−.77			−.89			.71
	VF						.99		−.78			−.90		.70
d_c	MJ								.87		.90			−.71
	VF											.93		−.71
d_s	MJ												.75	−.74
	VF												.75	−.72

**PHOLP = physical overlap, d_c = choice model distance ($-\log \eta_{ij}$), d_s = Torscal distance. See Table 2 for explanation of other headings.*

considerations suggest that concern with processing mechanisms and direct investigation of their properties may be more fruitful than subjecting complex data to multidimensional scaling programs. For instance, use of adaptation techniques with single features may help point to which features are lost during tachistoscopic exposure and whether they are lost in an all-or-none manner. That is, if a feature is adapted out prior to exposure and performance and confusions are not affected, then that feature was probably not present during exposure or was not needed for identification.

As in the group study, Pearson product-moment rs were obtained among analogous parameters and various measures of interletter distance. Tables 4 and 5 show these to be highly similar to those obtained with the group, the individual correlations being somewhat higher. Table 4 gives the correlations for the bias parameters for the three models for each S (VF, MJ); the correlations here tended to be low between Ss, between the all-or-none model and the other two, and higher within S and between the overlap and choice models. Thus, the two Ss seem to have somewhat different preferences for the letters, and the overlap and choice models appear to be describing the biases within S in a similar manner. The correlations of the individual biases with the group bias parameters (not shown in the table) were all less than .35 in absolute value, averaging .04.

Correlation coefficients for the similarity parameters and related measures are shown in Table 5. The relationship for parameters within model and across individuals and of individual with group are quite high (from .76 to .99), indicating that the models tended to be consistent for different individuals and for the group compared with the individuals. The correlation of choice with overlap parameters was less high but the positive relation here together with that for the bias parameters suggests that the models are explaining homologous properties of the data and that the lower correlation for the similarity parameters may be due to nonlinearities in the relationship.

This table also contains correlations between the similarity parameters, the scaled distances, and these and a measure of physical similarity obtained by superimposing all pairs of stimulus letters. These range from an absolute value of .54 to .93, the lowest values being the correlation of overlap parameters with physical overlap and the highest being the correlation of $-\ln(\eta_{ij})$ with the scaled distances. In this table, the correlations seem to be very consistent for the two Ss. Also, one could predict about .50 of the variance associated with the confusions as represented by the choice model similarity parameters on the scaled distances simply by superimposing templates of the letters on graph paper and measuring their overlap; possibly, some nonlinear transformation would make the correspondence even greater.

Abridging the results of the correlation analysis: (1) Ss confuse capital English alphabetic characters in much the same way but reveal different preferences or response biases when operating in the present kind of stimulus-degraded situation; (2) group data seem to be fairly accurate in describing individual similarity structure when using the choice or overlap models; (3) a simple measure of physical similarity is correlated better than .70 with distance and choice similarity measures; and (4) choice model similarity parameter correlations among individuals and of individuals with the group were especially high.

This study of individuals and the previous group study have attempted to provide an in-depth analysis for alphabetic confusion data obtained under tachistoscopic, near-threshold conditions. The analysis included a test of recognition models, multidimensional scaling of similarity parameters, and a correlational analysis of various model structures and distance and physical similarity measures. In summary: (1) The choice and overlap models were about equal to one another and superior to the all-or-none model in describing similarity and confusions, but the all-or-none model was better in predicting probability correct.[3] (2) The scaling results supported a Euclidean over a city-block metric and suggested that about four dimensions are needed for a description of the psychological space, using the "elbow" criterion (Kruskal, 1964a, b). However, the scaling programs were less successful in determining indisputable perceptual dimensions. (3) The overlap and choice models seemed to be explaining the data in an analogous manner and Ss appeared to exhibit like similarity structure. (4) A crude "template" measure of letter similarity was positively correlated with similarity and negatively correlated with distance measures. (5) For some purposes, group-averaged data may be sufficient to describe or predict individual similarity structure but not individual bias structure.

APPENDIX

Derivations of the confusion matrix formulae for the three models are given in the earlier paper (Townsend, 1971). Here, the theoretical predictions will simply be stated.

ALL-OR-NONE ACTIVATION MODEL

The confusion matrix probabilities for this model are given by

$$c_{ij} = (1 - \sigma_i)p_j \qquad i \neq j$$

and

$$c_{ii} = (1 - \sigma_i)p_i + \sigma_i, \quad i,j = 1, 2, \cdots, N,$$

where c_{ij} gives the probability of

responding "j," given stimulus "i," and there is the usual E-defined one-one mapping of responses to stimuli. The guessing bias is given by "g_i" and the probability of perfect perception by "σ_i." "N" is the number of stimuli and responses.

OVERLAP ACTIVATION MODEL

Here, the confusion values are

$$c_{ij} = \xi_{ij} \cdot \frac{g_j}{g_i + g_j} \quad i \neq j,$$

and

$$c_{ii} = \xi_{ii} + \sum_{k \neq i}^{N} \xi_{ik} \frac{g_i}{g_i + g_k},$$

$$i,j = 1, 2, \cdots, N,$$

where N is the number of symbols in the alphabet, 26 here. ξ_{ij} is the probability of confusion of stimulus "j" with stimulus "i" ($\xi_{ij} = \xi_{ji}$), ξ_{ii} refers again to perfect perception of stimulus "i," and $g_i/(g_i + g_k)$ is the probability of responding "i," given a confusion of "i" and "k."

CHOICE MODEL

Like the overlap model, the choice model is capable of representing stimulus similarity:

$$c_{ij} = \frac{\eta_{ij}\beta_j}{\sum_{k=1}^{N} \eta_{ik}\beta_k} \quad \text{for all } i,j = 1, 2, \cdots, N,$$

where η_{ij} is a parameter reflecting stimulus similarity and β_j reflects response strength.

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NOTES

1. The complete theoretical confusion matrices are deposited with the National Documentation Institute.

2. For example, it may be appropriate to employ a representative surface in a multidimensional space upon which geodesics may be computed or to consider the psychological dimensions as subtending angles greater or less than 90 deg.

3. A maximum likelihood estimation technique suggested by J. E. K. Smith (personal communication, 1970, University of Michigan) is planned for application to the individual and group data and may yield different and/or better fits than the present analysis.

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