MULTIDIMENSIONAL MODELS OF PERCEPTION AND COGNITION

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8 Signal Detection Analyses of Dimensional Interactions

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INTRODUCTION

This chapter traces the development and impact of Signal Detection Theory in the study of perception. Emphasis is on the extension of Signal Detection Theory to visual perception in two-dimensional perceptual spaces, although the results presented here are general and not restricted to the visual domain or the two-dimensional case. Within the context of General Recognition Theory (GRT; Ashby & Townsend, 1986; see also chaps. 6 and 16 in this volume), theoretical relationships exist between the unobservable notions of perceptual separability and perceptual independence and the observable (and estimable) concepts of sampling independence, marginal response invariance, and two sets of signal detection parameters (Kadlec & Townsend, 1992). These relationships and their applications will be presented.

A procedure for testing perceptual independence and perceptual and decisional separabilities follows from these theoretical results. We illustrate it with three sets of data. The first data set is a direct application of the results in a two-dimensional space where the stimuli have two components each at one of two possible levels of presentation. The second data set illustrates the generalization to a situation where stimuli are composed of four components each at one of two possible levels. The third data set generalizes the procedure to a case where stimuli are two-dimensional, but each component has one of four possible levels of presentation. Some issues related to these generalizations of Signal Detection Theory to more than one and two dimensions will be discussed.

The organization of the chapter is as follows. In the first section we briefly review unidimensional Signal Detection Theory and some of its applications and

extensions. In the second section we discuss perceptual dimensions, the definitions, and some ways in which they have been studied. In the third section, the GRT framework is briefly reviewed, and two sets of signal detection parameters based on generalization of signal detection theory to the multidimensional case are introduced. The various notions of "perceptual independence" are explicitly defined within the GRT framework in the fourth section, and in the next section the major results of Ashby and Townsend (1986) and Kadlec and Townsend (1992) relating various notions of "perceptual independence" with each other and the two sets of signal detection parameters are presented. (Note that we use "perceptual independence" in quotation marks to refer to the general sense of independence as used in the literature with its various connotations, and we reserve perceptual independence without quotation marks for the notion precisely defined in the fourth section.) These theoretical results lead to a method of testing perceptual separability and independence, which is presented and illustrated with three sets of data in the sixth section. In the final section of the chapter, some future directions and extensions are discussed. We attempted to make each section self-contained; thus, readers familiar with the material in any of the sections may skip to the next.

SIGNAL DETECTION THEORY AND ITS IMPACT ON THE STUDY OF PERCEPTION

Historically, Signal Detection Theory stemmed from the direct application of Statistical Decision Theory to the detection of auditory signals embedded in noise (Peterson & Birdsall, 1953). It was immediately recognized that this approach could be easily applied to other sensory domains, particularly the detection of visual stimuli (Tanner & Swets, 1954; see also Green & Swets, 1974), and this unidimensional approach could be generalized to the recognition of more than one stimulus, for example, two different auditory stimuli or color stimuli (Tanner, 1956), and the detection of various components of complex auditory signals (Green, 1958).

The traditional experimental paradigm was a simple judgment task where on a given trial the observer responded yes or no, depending on whether he or she detected a signal embedded in some noise that could potentially mask the true signal. Many such trials (usually hundreds) would be presented to obtain estimates of two probabilities: the probability of a *hit* calculated as the proportion of trials on which the observer correctly responded that the signal was present, and the probability of a *false alarm* estimated as the proportion of trials on which the observer incorrectly reported that he or she detected the signal when it was not presented.

The basic underlying assumption is that over trials a given stimulus (whether a signal in noise or noise alone) produces a distribution of perceptual effects, denoted here as $g_s(x)$ and $g_n(x)$ for the signal + noise and noise-alone distributions, respectively. These underlying distributions, one for each type of trial, are then used along with the estimated probabilities of hits and false alarms to calculate the sensitivity estimate d' and an estimate of the response bias β from the following relations:

$$P(\text{hit}) = P(\text{signal reported} \mid \text{signal present}) = \int_c^{\infty} g_s(x)\, dx$$

and

$$P(\text{false alarm}) = P(\text{signal reported} \mid \text{signal absent}) = \int_c^{\infty} g_n(x)\, dx\ ,$$

where c is the decision criterion such that if the perceptual effect x falls above c the observer responds that he or she detected the signal, and if the perceptual effect falls below c the observer responds that he or she did not detect the signal. The two parameters are defined as follows: the d', or sensitivity parameter, represents the (standardized) distance between the means of the noise and the signal + noise densities,

$$d' = \frac{\mu_s - \mu_n}{\sigma_n}\ ,$$

and β, the response bias parameter, is the ratio of the two densities at the criterion point c,

$$\beta = \frac{g_s(c)}{g_n(c)}\ .$$

The usual assumptions are that both densities are normal and that the noise distribution has a mean of 0 ($\mu_n = 0$) and standard deviation of 1 ($\sigma_n = 1$).

Since the introduction of Signal Detection Theory, its applications have flourished in various research domains. These research areas will not be reviewed here, but to give the reader a flavor of the variety of applications, we give two examples: (a) multimodal signal detection was used to evaluate the degree to which the visual and auditory perceptual systems interact and share capacity (e.g., Eijkman & Vendrik, 1965; Fidell, 1970; Taylor, Lindsay, & Forbes, 1967); and (b) examination of selective influence of various experimental factors on the two (d' and β) parameters, for example, stimulus intensity on the sensitivity parameter d', and payoffs or cost effects on the response bias parameter β. The typical experimental paradigms in which Signal Detection Theory has been used include detection of stimuli presented very briefly or at sensory threshold using yes-no judgment tasks or two- or four-interval forced-choice tasks, and discrimination between (or identification of) highly confusable stimuli.

A third example, and one that is directly relevant to our discussion, involves application of Signal Detection Theory to evaluate "independence" (sometimes also referred to as separability or orthogonality) of perceptual dimensions representing the stimulus components in multiattribute stimuli. This includes studies on whether the "processing channels" for each of the stimulus components operate independently in some sense. Two examples in the auditory domain involve questions of whether the components of two-dimensional stimuli (e.g., two tones at different frequencies presented simultaneously) can be recognized "independently" of each other (e.g., Corcoran, 1967), and whether the perceptual dimensions of pitch and loudness are "independent" (e.g., Zagorski, 1975). Similarly in vision, Signal Detection Theory has been employed to examine whether the various spatial frequency "channels" operate "independently" (e.g., Graham, 1989; Hirsch, Hylton, & Graham, 1982; Olzak, 1986).

Most studies that evaluate the "independence" of stimulus components in multiattribute stimuli employ a form of *feature-complete factorial designs*. In these designs, the stimulus set is formed by combining each level of each component factorially with every level of every other component. Ashby and Townsend (1986) called such designs *complete identification* designs. When confidence judgments on each component are required with such a stimulus set, it is called the *concurrent experiment* (e.g., Ashby, 1988; Graham, 1989; Olzak, 1986). In each of these designs, performance on a given component at a fixed level is then compared across levels of the other component(s).

The various applications of Signal Detection Theory were paralleled by some theoretical developments, for example, to the detection of a number of equally detectable orthogonal signals (Green & Birdsall, 1978; Nolte & Jaarsma, 1967). However, little in the way of formal development of multidimensional Signal Detection Theory was accomplished. This could primarily be due to the problem of generalizing the decision criteria of Signal Detection Theory to two- and higher-dimensional perceptual space (see end of the third section on GRT).

PERCEPTUAL DIMENSIONS AND THEIR "SEPARABILITY"

Around the same time as Signal Detection Theory was introduced in the study of psychophysics (mid-1960s to early 1970s), psychologists studying perception began to devote attention to the issue of whether and which perceptual dimensions are perceived "independently" using various response time paradigms. Based on earlier work by Attneave (1950), Garner and his co-workers, as well as others since then (e.g., Burns & Shepp, 1988; Melara & Marks, 1990a, 1990b; Shepard, 1964), have been interested in determining how a number (usually two) of different stimulus components are perceived when viewed in combination with each other in the same stimulus. In general, if one component does not have any influence on the perception of the other, the components are said to be separable (Garner, 1974); conversely, if the perception of one component is influenced by the other, the two are said to be integral (Lockhead, 1966). Much earlier work in this area is summarized in Garner (1974). (See also chap. 7.)

In most of this work, the experimental paradigms used response times as the dependent measure and included speeded classification tasks (e.g., Cheng & Pachella, 1984; Lockhead & King, 1977; Melara & Marks, 1990a, 1990b), same-different judgment tasks (e.g., Dixon & Just, 1978), and absolute judgment tasks (e.g., Weintraub, 1971). Taken together, these paradigms were to provide converging evidence about the separability or integrality of numerous perceptual dimensions.

Separability in Garner's sense is defined operationally without much theoretical basis. This has sometimes resulted in debates about what separability and integrality mean. For example, some investigators believe that separability and integrality are not dichotomous, but represent two extremes of a continuum (e.g., Cheng & Pachella, 1984; Grau & Kemler-Nelson, 1988; Smith & Kemler, 1978). Within such a continuum, Cheng and Pachella (1984) interpreted separability in terms of a correspondence of physical dimensions of a stimulus to a finite number of psychological attributes that can be selectively attended to; those physical dimensions that have such a correspondence to a psychological attribute are defined as separable. Lockhead and his co-workers (Lockhead & King, 1977; Monahan & Lockhead, 1977), on the other hand, have claimed that performance on classification and identification of integral stimuli can best be predicted based on psychological distances between stimuli in a similarity space. In fact, the traditional view was that stimuli composed of integral dimensions are not (psychologically) perceived as dimensions at all, but rather as dimensionless "blobs" (Garner, 1974; Lockhead, 1979; but see Melara & Marks, 1990a). It is difficult to resolve such definitional problems in a theory-free perspective.

Within GRT, perceptual separability has a clearly defined meaning (see section on Various Concepts of "Perceptual Independence"). Although perceptual separability is defined in somewhat different terms in the two approaches, there is hope that at least some of the psychological substrate that is pertinent to the Garner train of research will overlap with that of Ashby and Townsend (1986). Ashby and Maddox (see chap. 7) have begun to link Garner's definition of separability with the theoretical notion of perceptual separability defined in GRT. More evidence on this question, however, is critically needed.

The method for testing perceptual separability based on multidimensional Signal Detection Theory that we present in this chapter involves an identification-discrimination paradigm with accuracy measures as the dependent variable, thus providing another line of investigation on perceptual separability. Whereas Multidimensional Scaling and Factor Analysis are methods specifically designed to uncover underlying psychological or perceptual dimensions, investigators studying perceptual separability in multiattribute stimuli typically assume, or concretely specify, what the perceptual dimensions are. Once established by the investigator, the question then becomes whether these dimensions interact, and if so how. The method based on the GRT perspective developed here is of the latter genre. It is more general, however, as GRT encompasses and unifies many different paradigms, such as perceived similarity (Ashby & Perrin, 1988), preference (see chap. 6), and a number of models of decision making (Ashby & Gott, 1988). Additionally, our hope is that the method we present will be extended to make it useful also for identifying perceptual dimensions. Results of new tests and modeling may thus be able to provide support for, or refute, existing notions of what constitutes a given set of perceptual dimensions. A method that could serve both of these functions, to find the relevant perceptual dimensions and to examine how they are perceived, would be very useful and would provide an interesting alternative to Multidimensional Scaling.

Perhaps the single most important issue when studying perception from this "dimensional" perspective concerns the definition of perceptual dimensions. Tversky and Krantz (1970) have defined perceptual dimensions as the organizing principles or factors along which stimuli are perceived and structured. These perceptual dimensions are contrasted with physical attributes that characterize the stimuli or dimensions derived from Factor Analysis or Multidimensional Scaling. For example, physical attributes (or physical dimensions) may be stimulus intensity, hue, or line orientation for visual stimuli, and frequency and amplitude of sound waves for auditory stimuli. A perceptual dimension, on the other hand, may be the strength with which a given feature is represented in the perceptual space (e.g., Wickelgren, 1967), possibly realized in the nervous system by the frequency of firing of neurons or the number of neurons firing in a specific cortical area. This is the sense of perceptual dimension that we employ here. Tversky and Krantz (1970), however, assume that perceptual dimensions are by definition "independent", which we do not, since one of the goals of our investigation is to specifically examine their "independence" (in the various senses of the word, as defined later).

The identification of perceptual dimensions in the different sensory domains has met with mixed success. In the case of audition, the perceptual dimensions of loudness, pitch, and timbre correspond best to the physical stimulus dimensions of amplitude, frequency, and waveform shape, respectively, of a sound wave. (Note that there does not necessarily need to be a one-to-one mapping between physical and perceptual values.) In two recent studies, Melara and Marks (1990a, 1990b) introduced two interesting distinctions among perceptual dimensions for auditory stimuli, although presumably these would also hold for other senses. The first involves a distinction between "primary" versus "nonprimary" perceptual dimensions (Melara & Marks, 1990a). It employs an experimental paradigm where the axes of the physical dimensions, from which the compound stimuli are constructed, are experimentally rotated. That orientation that results in separability in Garner's sense, and thus in dimensions that are perceived as the most salient without suffering performance deficits, then corresponds to the "primary" dimensions (Melara & Marks, 1990a).

The second distinction (Melara & Marks, 1990b), "hard" versus "soft" dimensions, concerns the contextual effects of one dimension on the other. The assertion is that in a feature-complete factorial design, two types of contexts operate simultaneously to affect perception of the individual components. In classifying stimuli on one dimension, *intraclass context* is defined as variation along an unattended dimension that interferes with classification on the attended dimension because the attended dimension itself may now be perceived as varying. In classification of pairs of stimuli when the two dimensions are correlated, for example, classifying stimuli that are "high" (or "low") on both dimensions (in this case positively correlated dimensions), this *redundant context* may make the stimuli more discriminable and thereby facilitate performance. Melara and Marks found that auditory perceptual dimensions can be distinguished on the basis of effects of intraclass context; pitch and timbre are "hard" perceptual dimensions because they resist intraclass context and profit from redundant context, whereas loudness is a "soft" perceptual dimension since both contexts affect its perception.

Both of these distinctions are interesting extensions of Garner's research and enhance our understanding of perceptual dimensions. As discussed before, this work was primarily done in the auditory domain, since auditory perceptual dimensions seem to be more clearly specified. Applying such paradigms to other senses should provide many fruitful insights.

In vision, the picture for perceptual dimensions seems more complex. Various "modules" for processing different aspects of visual perception, such as motion, color, texture, depth, and contour (or form), have been proposed, and questions have been raised whether these are or can be processed "independently" (e.g., Livingstone & Hubel, 1988; Marr, 1982; Nakayama & Silverman, 1986; Treisman & Gelade, 1980; Treisman & Gormican, 1988). The dimensions for some of these modules have been proposed and effectively studied. For example, color perception involves the three perceptual dimensions brightness, hue, and saturation, which correspond to the physical stimulus attributes amplitude, frequency (or wavelength), and purity of light waves, respectively (e.g., Burns & Shepp, 1988; Krantz, 1974). For texture perception, Julesz (1981, 1985) has an extensive theory based on what he calls "textons" whose first- and higher-order statistical characteristics have different perceptual qualities.

For pattern or form perception, however, there is no general agreement on what the basic perceptual dimensions are. A classic example, involving dissimilarity ratings of rectangles, illustrates this problem of defining the relevant perceptual dimensions. Whereas some subjects judge dissimilarity of rectangles based on the dimensions of height and width, others use the dimensions of area

and shape (e.g., Krantz & Tversky, 1975; Lazarte & Schönemann, 1991). Note that this may be a decisional and not necessarily perceptual effect; nevertheless the question of how to define the relevant perceptual dimensions remains.

The discovery of cortical columns in visual areas of the brain that preferentially respond to lines and edges of specific orientations (e.g., Hubel, 1982; Hubel & Livingstone, 1985; Hubel & Weisel, 1959, 1968) led to speculation that these simple orientation lines and edges could represent the perceptual dimensions of which all patterns were subsequently composed. Other investigators, however, argue that the brain analyzes a visual scene in terms of spatial frequency components contained in the scene, and that each frequency component is subserved by its own processing "channel" (e.g., De Valois & De Valois, 1988 Graham, 1981, 1989; Watson, 1983). Evidence from primates also seems tc support this latter view; visual cortical cells do preferentially respond to differen spatial frequencies (e.g., K. De Valois & Tootell, 1983; R. De Valois, Albrecht & Thorell, 1982).

Some studies have attempted to test these two alternative hypotheses—whethe bars and edges or spatial frequency components are the relevant physiologica stimuli. The results suggest that spatial frequency information is physiologicall more relevant than the more simple (but spectrally much more complex) bars an edges (see, e.g., De Valois & De Valois, 1988, pp. 206–211). The two views however, have not yet been resolved to everyone's satisfaction, and they may nc be antithetic, since neural responses depend on frequency and orientation of th stimulus, as well as other factors such as motion, direction of motion, and en stopping (R. De Valois, Yund, & Hepler, 1982; Webster & R. De Valois, 1985). I is also a big leap to suggest that perceptual dimensions are the same as the stimulu attributes that drive *single* cortical cells. In order to specify the relevant perceptu dimensions, much work is yet needed on all fronts and particularly with respect t merging "top-down" approaches of perception with the "bottom-up" approach c physiology.

GENERAL RECOGNITION THEORY AND SIGNAL DETECTION MACRO- AND MICROANALYSES

With the different experimental paradigms employed to investigate interactio of perceptual dimensions, the terminology regarding "perceptual independenc of dimensions has become quite perplexing. Within the context of GRT, how ever, Ashby and Townsend (1986) defined and interrelated many of the vario terms that now connote "perceptual independence"; these definitions will l presented in the next section. In this section, we briefly review GRT and intr duce the two sets of parameters of signal detection macroanalyses and micr analyses. (See chaps. 6, 7, and 16 for other results and extensions of GRT

In the notation of Chapter 1, in GRT a stimulus that is constructed from tv components is represented as A_iB_j, i = 1, . . . , n, j = 1, . . . , m, where A and B denote the (physical) stimulus components (e.g., intensity, line length, etc.), each of which is represented by a dimension in the perceptual space (denoted X and Y, respectively). More generally, a stimulus may have more than two components, for example, components A, B, . . . , N, each represented by a perceptual dimension and each with its own levels. The subscripts represent the levels of the corresponding components in the presented stimulus. It has sometimes been mistakenly assumed that when only two levels are included in the design they imply feature presence or absence. In fact, Garner and Haun (1978) defined "features" as perceptual dimensions that are either present or absent. However, it is worth stressing that in our usage the component or feature levels can take on *any* values, although we sometimes use feature presence and absence in our examples.

GRT views perception as a two-stage process. When a stimulus A_iB_j is presented, perceptual processes of the system operate on it to produce a perceptual effect (x,y). Based on this effect, the decisional processes then choose the response that the system will make. The theory assumes that the perceptual effect of any given stimulus (specifically of each component of the stimulus) is not constant over trials but has some (usually continuous) distribution. This allows us to represent each stimulus by a joint probability density of the perceptual effects of the constituent features in the perceptual space. In the case where there are two stimulus components, a bivariate density, $f_{ij}(x,y)$, represents the distribution of perceptual effects of stimulus A_iB_j. For example, in a feature-complete factorial design with two components each with two levels (e.g., $i = 1, 2$ and $j = 1, 2$ with 1 denoting feature absence and 2 denoting feature presence), we have four bivariate densities in the perceptual space: one for the blank stimulus, one for each of the stimuli composed of each feature individually, and one for the stimulus containing both features. Equal probability contours representing these four stimuli are shown as circles and ellipses in the upper left part of Figure 8.1 (the other parts of this figure will be explained in more detail later).

For the decision process, it is assumed that the observer sets up decision criteria that divide the perceptual space into different response regions. Each component usually has its own criterion (lines labeled c_A and c_B in Figure 8.1). A stimulus presentation on a given trial results in a percept represented by a single point in this perceptual space. The subject's response then corresponds to the region in the perceptual space where the percept falls in relation to the decision criterion for each component. For example, if the percept falls above the criterion for component A (c_A on Figure 8.1) and below the criterion for component B (c_B), the subject responds that feature A was present and feature B was absent. Stated this way, GRT is completely general and does not assume any particular form for the densities. We will, however, employ the Gaussian version of GRT and estimate signal detection parameters based on Gaussian densities.

Each joint density has associated with it its corresponding marginal densities.

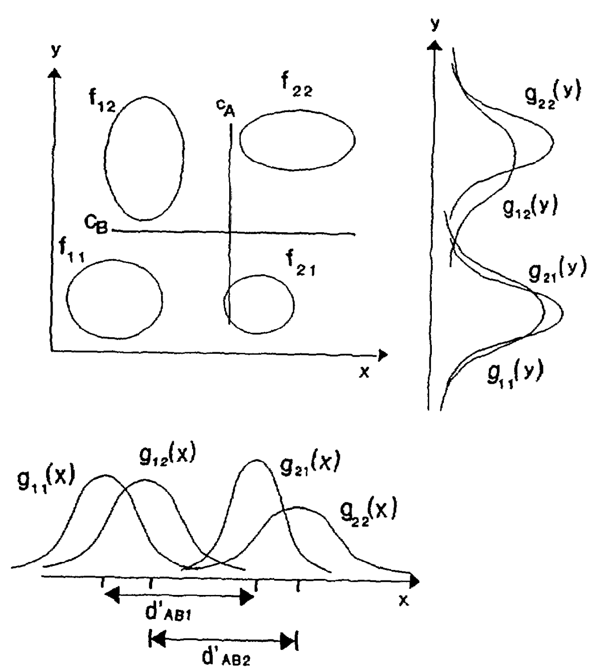

FIG. 8.1. Equal-density contours of the joint densities representing the perceptual effects of four stimuli constructed from two dimensions, with two levels on each dimension (upper left). For each stimulus, marginal densities, $g_{ij}(x)$ (bottom of figure) and $g_{ij}(y)$ (right-hand side), result after integrating out Y and X, respectively. These marginal densities are then used for computing the marginal d''s and β's. See text for further explanation.

In the two-dimensional case, for each stimulus A_iB_j we have one marginal density for the perceptual effect of component A (X), denoted by $g_{ij}(x)$ (see bottom of Figure 8.1), and a marginal density for Y (component B), $g_{ij}(y)$ (see the right side of Figure 8.1). According to this multidimensional point of view, two sets of signal detection parameters can be defined (Townsend, Hu, and Evans, 1984; Townsend, Hu, and Kadlec, 1988). The mathematical definitions are given in Appendix A.

Signal detection macroanalyses use the marginal densities, resulting in d''s and β's for each component, at each level of the other component. For example, in the four-stimulus case in Figure 8.1, we have one d' and β for component **A** at the first level of component **B**, denoted d'_{AB1}, and one d' and β at the second level of component **B**, d'_{AB2} (see bottom of Figure 8.1). The d''s and β's for component **B** at each of the levels of component **A** are defined analogously and would appear along the y-axis. Investigations of perceptual "independence" using Signal Detection Theory have usually employed these *marginal* estimates (e.g., Graham, Kramer, & Haber, 1985; Olzak, 1986; Wickelgren, 1967).

The second set of signal detection parameters are obtained from *signal detection microanalyses* (Townsend et al., 1984, 1988). These *conditional* signal detection parameters are obtained for each component at each level of another component, *within the same stimulus*, by conditioning on how the other component in the stimulus was perceived. In the two-dimensional example, two pairs of parameters are obtained for each component: (a) by conditioning on whether the second component was a "hit" (correctly reported as present) or a "miss" (incorrectly reported as absent), both of which imply that the second component was present in the stimulus, and (b) by conditioning on whether the second component was a "correct rejection" (correct report that it was absent) or a "false alarm" (incorrect report that it was present), which indicates that the second component was not in the stimulus. Please note that we keep this nomenclature ("hits", "miss", etc.) even in the more general case when there are more than two levels of each component, even though these terms will then actually stand for conditioning above or below a given criterion. These conditional estimates were employed by Sorkin, Pohlmann, and Gilliom (1973) in an innovative feature-complete factorial design with two auditory stimuli.

The conditional parameters for a given component are based on the densities that result from the joint densities after we condition on the particular response made for the other component (see Figure 8.2 and Appendix A). Consider the d' for component **A** conditioned on the hit of component **B**, d'(**A**|**B** hit), in our binary-valued two-dimensional example. The hit of **B** implies that **B** was present in the stimulus; thus, we use those portions of $f_{12}(x, y)$ and $f_{22}(x, y)$ that fall *above the c_B criterion* to obtain d'(**A**|**B** hit). To obtain d'(**A**|**B** miss), we use those portions of $f_{12}(x, y)$ and $f_{22}(x, y)$ falling *below* the c_B criterion.

A critical problem in generalizing Signal Detection Theory to two or more dimensions is the generalization of the response bias parameter β when a decision criterion is not parallel to the axis. In one dimension, the criterion is a point on the axis representing the perceptual effect such that, if the percept on a given trial falls above the criterion, the decision is to report the feature (signal) as present, and if the percept falls below the criterion it is reported as absent. In two dimensions, however, the criterion becomes a line or, more generally, a curve. When the criterion, say c_A, is not parallel to the perceptual y-axis, there does not exist a unique point to represent the "average" criterion for **A** such that the

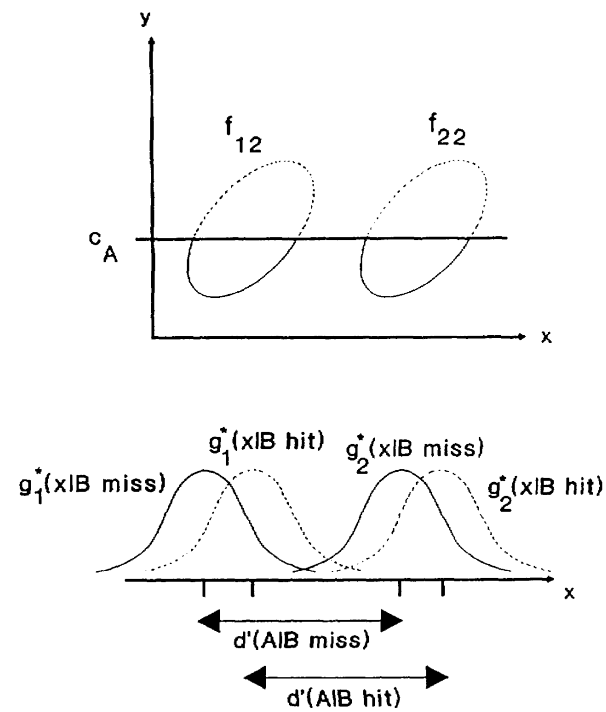

FIG. 8.2. Illustration of conditional d''s. Joint densities for only two stimuli, with feature **B** at the second level, are shown (upper figure); after normalization, the dashed portions of the contours, above the c_B criterion, are used to obtain marginal densities for computing d'(**A**|**B** hit), and the solid portions give d'(**A**|**B** miss) (bottom figure).

probabilities of hits and false alarms are preserved when mapping from the two-dimensional space to one (the marginal x) dimension. The definition of β thus becomes more complex, since it would depend on how component **B** was perceived (it would be a function of y). Similar complexities plague the conditional densities on which the conditional parameters are based. In many of the theorems, we therefore assume parallel decision criteria.

VARIOUS CONCEPTS OF "PERCEPTUAL INDEPENDENCE"

Different notions connoting "perceptual independence" have been defined within the GRT framework (Ashby & Townsend, 1986). The three theoretical definitions we consider here are perceptual independence, perceptual separability, and decisional separability. Two additional concepts of "independence", sampling independence and marginal response invariance, are empirically observable conditions that are closely related to the former ones.

To make these definitions more explicit, consider the following example. Suppose the stimulus set is composed of a feature-complete factorial combination of (some of) the following components and levels: component **A**—geometric shapes (circles, squares, triangles); component **B**—colors (red, blue, green); component **C**—sizes (small, large); and component **D**—brightness (dim, bright).[1] The simplest feature-complete factorial design with components **A** and **B**, each with two levels, has four stimuli in the ensemble (red circle, blue circle, red square, and blue square), and each stimulus is represented by a bivariate density in a two-dimensional perceptual space.

For each concept, we first give the formal definition, explain it in terms of our previous example, and then discuss the generalizations to (a) two-dimensional cases with more than two levels for each component, and (b) higher-dimensional cases. We begin with the theoretical definitions.

Perceptual Independence

Definition 8.1: Perceptual independence (this is Definition 1.1) of components **A** and **B** in a feature-complete factorial design holds in stimulus A_iB_j if and only if the perceptual effects of **A** and **B** are statistically independent, that is, if and only if

$$f_{ij}(x, y) = g_{ij}(x)g_{ij}(y), \qquad \text{for all } x \text{ and } y\ .$$

This condition signifies that within a given stimulus, say a blue square, the perceptual effect of one component (blue) does not influence the perceptual effect of the other component (square), no matter where in the perceptual space the overall percept may fall. Thus, on every trial where a given stimulus is presented, if perceptual independence holds in that stimulus one component will not affect the other. Note that for Gaussian densities, this amounts to a zero correlation between the perceptual effects of the two components.

Generalizing this definition to more than two levels on each of two components is straightforward: simply replace the corresponding values for i and j. In this two-dimensional case with m levels on each dimension, Ashby (1988) has shown how a generalization of the tetrachoric r can be employed to test for perceptual independence.

When more than two dimensions are present, the generalization becomes more interesting, since alternative definitions exist. We define *mutual* perceptual independence of *all* components simultaneously. For example, in the four-dimensional case, mutual perceptual independence of all four components in stimulus $A_iB_jC_pD_q$ holds when

$$f_{ijpq}(x, y, z, w) = g_{ijpq}(x)g_{ijpq}(y)g_{ijpq}(z)g_{ijpq}(w)$$

for all x, y, z, and w.

Alternatively, *joint* (pairwise) perceptual independence is defined for pairs of components at each *fixed* combination of levels of the other component(s), essentially conditioning on the levels of the other component(s). For example, in stimulus $A_iB_jC_pD_q$, pairwise joint independence of components **A** and **B**, at given values of Z and W, representing the perceptual effects of components **C** and **D**, respectively, holds when

$$f_{ijpq}(x, y|Z=z, W=w) = g_{ijpq}(x|Z=z, W=w)g_{ijpq}(y|Z=z, W=w)$$

for all x and y, where $g_{ijpq}(x|Z=z, W=w)$, for example, is the marginal density of the *three* perceptual effects X, Z, and W, with Z fixed at z and W fixed at w. More concretely, shape (**A**) and color (**B**) are jointly pairwise independent in a given stimulus, but only at particular values of Z and W, that is, only when the percept of the stimulus falls in a particular region of the perceptual space on the **C** (size) and **D** (brightness) dimensions (e.g., when it appears larger and brighter, but not when it appears smaller and dimmer). Note that mutual perceptual independence of all components implies the joint perceptual independence of the subsets of components.

Similarly, for four or more components, we can also define joint three-way independence between, say, components **A**, **B**, and **D** at a given value of Z representing the perceptual effect associated with component **C** in the given stimulus. Here we have

$$f_{ijpq}(x, y, w|Z=z) = g_{ijpq}(x|Z=z)g_{ijpq}(y|Z=z)g_{ijpq}(w|Z=z)\ ,$$

for all x, y, and w, where $g_{ijpq}(x|Z=z)$, for example, is the marginal density of the *two* perceptual effects X and Z, with Z fixed at z. In this example, the components of shape, color, and brightness are jointly (three-way) independent, but depend on the particular value of Z (size).

These definitions clearly indicate that perceptual independence is concerned with *within*-stimulus effects. Thus, it may be that in the same experiment perceptual independence of the components may hold in one stimulus (e.g., in the blue square) but not in another (e.g., the red square). Perceptual and decisional

[1]These stimulus components are chosen merely for illustrative purposes. The fact that each component may itself have multidimensional perceptual representations should not cause confusion.

separabilities, on the other hand, describe the effects that the components have on each other *across* different stimuli. These definitions are given next.

Perceptual and Decisional Separabilities

Definition 8.2: In a feature-complete factorial design with stimuli A_iB_j, i = 1,2, j = 1,2, component **A** is *perceptually separable* from component **B** if and only if the perceptual effect of component **A** does not depend on the level of component **B**—that is, if and only if

$$g_{i1}(x) = g_{i2}(x), \quad i = 1,2 .$$

Similarly, component **B** is *perceptually separable* from component **A** if and only if the perceptual effect of component **B** does not depend on the level component **A**—that is, if and only if

$$g_{1j}(y) = g_{2j}(y), \quad j = 1,2$$

Definition 8.3: In a feature-complete factorial design with stimuli A_iB_j, i = 1,2, j = 1,2, component **A** *(B) is decisionally separable* if and only if the decision about component **A** (**B**) does not depend on the level of component **B** (**A**)—that is, if the decision bound for component **A** (**B**) in the GRT representation is parallel to the *Y*-axis (*X*-axis).

In our two-dimensional example, if shape is perceptually separable from color, it means that when we average across colors the two densities for shape will be identical. In other words, the perceptual effect of the square will have the same distribution (across trials) whether it is obtained from the red-square stimulus trials or from the blue-square stimulus, and similarly for the perceptual effects of the circle stimuli. In yet other words, the level of the color component has no effect on the perception of the shape across trials. The same interpretation is given to decisional separability of one component across levels of the other.

In the definitions of perceptual and decisional separabilities, note that one feature may be perceptually or decisionally separable from the other while the other need not be perceptually or decisionally separable from the first. This is an important point, since in some of our theorems separability of one feature is sufficient, whereas in other theorems both components must be separable.

To generalize perceptual separability to more than two levels is straightforward. The requirement is that all marginal densities for each level of a given component be identical across *all* levels of the other component; for example, for components **A** and **B** with n and m levels, respectively, component **A** is perceptually separable from **B** when

$$g_{i1}(x) = g_{i2}(x) = \cdots = g_{im}(x), \quad \text{for } i = 1,2, \cdots, n ,$$

and component **B** is perceptually separable from **A** when

$$g_{1j}(y) = g_{2j}(y) = \cdots = g_{nj}(y), \quad \text{for } j = 1,2, \cdots, m .$$

In other words, the level on one component does not affect how the other component is perceived.

For more than two dimensions, we again have alternative definitions: *joint* perceptual separability and single-component perceptual separability. A pair of components, say **A** and **B**, are *jointly* (pairwise) perceptually separable from component **C** when the joint densities for perceiving components **A** and **B** are equivalent across levels of **C**: that is,

$$g_{ij1}(x, y) = g_{ij2}(x, y), \quad \text{for } i = 1,2 \text{ and } j = 1,2 .$$

This means, for example, that the density for the blue square is identical whether it is large or small, and similarly for the other color-shape combinations.

Single-component perceptual separability, for example for component **A** (from components **B** and **C**), holds when

$$g_{i11}(x) = g_{i12}(x) = g_{i21}(x) = g_{i22}(x), \quad \text{for } i = 1,2 .$$

Note that equality of the first two terms alone indicates separability of **A** across **C** at the first level of **B**, whereas equality of the third and fourth terms indicates separability of **A** across **C** at the second level of **B**. Similarly, equality of the first and third terms signifies separability of **A** across **B** at the first level of **C**, and the second and fourth terms denote separability of **A** across **B** at the second level of **C**. Single-component perceptual separability, even for all components, does not imply joint (pairwise) perceptual separability, nor vice versa, since different marginal densities are involved in the two types of definitions.

Generalizing decisional separability to more levels, we simply require that all the decision bounds, between all the levels on each dimension, be parallel to the perceptual axes. In higher-dimensional spaces, each of the decision bounds (in general now hyperplanes) must be parallel to its perceptual axis.

These then are the theoretical notions of "independence" defined within the GRT framework. The following two senses of independence are operational definitions. Both concern overall probabilities (corresponding to areas and volumes under the densities in GRT) rather than conditions that must hold at all perceptual values (x,y), and, thus, in this sense are weaker than the theoretical concepts defined before. Sampling independence is the global analog of perceptual independence and is a within-stimulus outcome. Marginal response invariance is analogous to perceptual separability.

Sampling Independence

Definition 8.4: If *a* and *b* denote the events that components **A** and **B**, respectively, are reported, then *sampling independence* in stimulus A_iB_j holds if and only if

$$P(a_2b_2|A_iB_j) = P(\text{A is sampled }|A_iB_j) \times P(\text{B is sampled }|A_iB_j)$$
$$= \{P(a_2b_1|A_iB_j) + P(a_2b_2|A_iB_j)\} \times \{P(a_1b_2|A_iB_j) + P(a_2b_2|A_iB_j)\}\ .$$

In other words, sampling independence holds in stimulus A_iB_j if and only if the probability that both features are reported is equal to the probability that feature A is reported (regardless of the level reported for the level of feature B) times the probability that feature B is reported (regardless of the level reported for feature A). This is simply that the joint probability of two events, a_2 and b_2, equals the product of the two marginal probabilities for each event.

In our example, this means that when a blue square, say, is presented to the observer, the probability that he or she will respond, for example, "red square" is equal to the probability of reporting "red" (regardless of shape or, in other words, the probability of reporting red squares plus the probability of reporting red circles) times the probability of reporting "square" (regardless of color, which is the probability of reporting red squares plus the probability of reporting blue squares). All other conditional response probabilities are defined similarly; that is, for each stimulus A_iB_j we can define $P(a_pb_q|A_iB_j)$, where the subscripts p and q can take on any of the values of i and j, respectively. In the previous example, on trials when blue squares were presented, sampling independences of the color and shape components are determined for red squares (as before), as well as for red circles, blue squares, and blue circles, by using this expression with the appropriate subscripts.

In the more general two-component case where there are n levels of component A and m levels of component B, sampling independence is defined as

$$P(a_2b_2|A_iB_j) = \left[\sum_{k=1}^{m} P(a_2b_k|A_iB_j)\right] \times \left[\sum_{k=1}^{n} P(a_kb_2|A_iB_j)\right]\ .$$

As before, this simply states that one joint probability equals the product of the two marginals, except now the marginal probabilities for each component level are computed across all the possible levels of the other component. To make this more concrete, consider components A and B each with three levels. On trials when blue squares, say, are presented, sampling independence of red squares holds if, as before, the probability of reporting red squares is equal to the probability of reporting red (now the probability of reporting red squares, red circles, plus red triangles) times the probability of reporting squares (red, blue, and green).

Generalizing sampling independence to cases where there are more than two components, we again define *joint* (pairwise) sampling independence for pairs of components at each *fixed* combination of levels of the other component(s). For example, in our three-dimensional scenario suppose we have joint pairwise sampling independence for components A (shape) and C (size), holding B (color)

constant, in stimulus $A_iB_jC_k$. Joint sampling independence of A and C holds (when red $\{b_1\}$ is reported) if the joint probability of reporting large (c_2) squares (a_2) equals the product of the marginal probabilities, or

$$P(a_2b_1c_2|A_iB_jC_k) = \left[\sum_{r=1}^{m} P(a_2b_1c_r|A_iB_jC_k)\right] \times \left[\sum_{p=1}^{n} P(a_pb_1c_2|A_iB_jC_k)\right]\ .$$

Note that b_1 is constant in this expression: We could obtain sampling independence of shape and size when the stimuli are red but not when they are blue. In four (and higher) dimensions, intermediate cases again arise. For example, three-way joint sampling independence is defined for three components as before, across the fixed levels of the fourth (and higher) component(s).

Mutual sampling independence is defined to hold when the joint probability of all components equals the product of all marginal probabilities. For example, in stimulus $A_iB_jC_k$, mutual sampling independence holds when the probability of reporting a large (c_2) red (b_1) square (a_2) equals the product of marginal probabilities of reporting large, of reporting square, and of reporting blue; that is,

$$P(a_2b_1c_2|A_iB_jC_k) = \left[\sum_{q=1}^{n}\sum_{r=1}^{s} P(a_2b_qc_r|A_iB_jC_k)\right] \times \left[\sum_{p=1}^{m}\sum_{r=1}^{s} P(a_pb_1c_r|A_iB_jC_k)\right] \times \left[\sum_{p=1}^{m}\sum_{q=1}^{n} P(a_pb_qc_2|A_iB_jC_k)\right]\ .$$

Since these are probabilities of events, mutual sampling independence of all components implies pairwise (and three-way, etc.) independence of the components (e.g., Hogg & Craig, 1978, p. 87). (In the Gaussian case, pairwise independence for *all* pairs also implies mutual independence.) This suggests a hierarchical testing procedure, beginning with the examination of mutual independence of all components and, if dependence is found working "down," determining where the dependencies originate.

Marginal Response Invariance

Definition 8.5: In a feature-complete factorial design with stimuli A_iB_j, $i = 1,2$, $j = 1,2$, *marginal response invariance* for feature A holds across the two levels of feature B if and only if

$$P(a_1b_1|A_1B_1) + P(a_1b_2|A_1B_1) = P(a_1b_1|A_1B_2) + P(a_1b_2|A_1B_2),$$
$$\text{for } i = 1,2 .$$

Similarly, marginal response invariance holds for feature **B** across the two levels of feature **A** if and only if

$$P(a_1b_j|A_1B_j) + P(a_2b_j|A_1B_j) = P(a_1b_j|A_2B_j) + P(a_2b_j|A_2B_j),$$
$$\text{for } j = 1,2 .$$

In other words, marginal response invariance holds for one component (say **A**) across the two levels of the second component (**B**) if and only if the probability of correctly recognizing component **A** does not depend on the level of component **B**. In terms of our hypothetical example, marginal response invariance for shape across colors means that the probability of correctly recognizing the shape as a square or a circle does not depend on the color of the stimulus.

To generalize marginal response invariance to more than two levels on two dimensions is again straightforward (see also Ashby, 1988). Marginal response invariance holds for component **A** (at level i) across levels of component **B** if

$$\sum_{j=1}^{n} P(a_ib_j|A_iB_1) = \sum_{j=1}^{n} P(a_ib_j|A_iB_2) = \cdots = \sum_{j=1}^{n} P(a_ib_j|A_iB_n),$$
$$\text{for } i = 1, \ldots, m;$$

and for component **B** (at level j) across levels of component **A** if

$$\sum_{i=1}^{m} P(a_ib_j|A_1B_j) = \sum_{i=1}^{m} P(a_ib_j|A_2B_j) = \cdots = \sum_{i=1}^{m} P(a_ib_j|A_mB_j),$$
$$\text{for } j = 1, \ldots, n;$$

Again, generalizations to more than two components result in two definitions. For example, in the three-dimensional case where each component has two levels, components **A** and **B** (at levels i and j, respectively) are *jointly* (pairwise) marginally invariant across levels of component **C** when

$$\sum_{k=1}^{2} P(a_ib_jc_k|A_iB_jC_1) = \sum_{k=1}^{2} P(a_ib_jc_k|A_iB_jC_2)$$
$$\text{for } i = 1,2 \text{ and } j = 1,2.$$

Another way of interpreting joint marginal response invariance is as the marginal response invariance of component **A** (at level i) across levels of component **C**, holding component **B** fixed at level j.

Single-component marginal response invariance is defined as the probability of correctly recognizing a given component regardless of the levels of *all other* components. For example, component **A** (at level i, for i = 1 and 2) is marginally invariant across components **B** and **C** if

$$\sum_{j=1}^{2}\sum_{k=1}^{2} P(a_ib_jc_k|A_iB_1C_1) = \sum_{j=1}^{2}\sum_{k=1}^{2} P(a_ib_jc_k|A_iB_1C_2)$$
$$= \sum_{j=1}^{2}\sum_{k=1}^{2} P(a_ib_jc_k|A_iB_2C_1)$$
$$= \sum_{j=1}^{2}\sum_{k=1}^{2} P(a_ib_jc_k|A_iB_2C_2) .$$

Unlike sampling and perceptual independence, however, joint marginal response invariance of all pairs of components does not imply single-component marginal response invariance, nor does single-component marginal response invariance for all components imply joint marginal response invariance for all pairs. Examples of confusion matrices where one holds and the other fails can be easily constructed. This also differs from conjoint measurement theory, where joint independence for all pairs of factors implies independence of all single factors (Krantz & Tversky, 1971).

In our present work, it seems more informative and relevant to use the more local, detailed (pairwise) definitions already given, which involve conditioning on the fixed levels of all other components. Essentially, we are interested in examining the separabilities of each perceptual dimension from every other dimension, and this may or may not depend on the levels of the third, or higher dimensions in the stimulus set. This latter way of generalizing is also consistent with our evaluations of the pairwise d''s in the multidimensional perceptual space. In other words, we are not so much interested here in the overall pooled effects of the other dimensions on a given dimension (i.e., single-component marginal response invariance or mutual perceptual or sampling independence) by evaluating one dimension without regard to the levels on the other dimensions.

THEORETICAL RESULTS FOR PERCEPTUAL AND DECISIONAL SEPARABILITIES AND PERCEPTUAL INDEPENDENCE

In this section we review the major theoretical results relating perceptual and decisional separabilities and perceptual independence with the observable (and testable) properties of sampling independence and marginal response invariance and the two sets of signal detection parameters. Within GRT, Ashby and Tow send (1986) related many of these concepts (as well as others) to each other and showed how some yield useful tests of perceptual separability and independence. We present three of their theorems here, as they are directly related to, and complement, our later results. The subsequent six theorems show the relationships among the signal detection macro- and microanalytic parameters with perceptual separability and independence (the proofs are given in Kadlec & Townsend, 1992). All theorems pertain to feature-complete factorial designs with two dimensions and two levels on each dimension; however, extensions of the results to higher-dimensional spaces and/or to more than two levels are direct (given the generalized definitions presented earlier). Where the result is stated only for component **A**, an analogous result also holds for component **B** simply by interchanging the roles of A and **B**.

General Theorems Relating Sampling Independence, Perceptual Independence, Perceptual Separability, Decisional Separability, and Marginal Response Invariance: Theorems 8.1–8.3

The first theorem relates sampling independence and perceptual independence (Theorem 1 in Ashby & Townsend, 1986).

Theorem 8.1: 1. Sampling independence of components **A** and **B** in stimulus $\mathbf{A}_i\mathbf{B}_j$ occurs if perceptual independence and decisional separability hold for both components.

2. *Perceptual independence of components A and B in stimulus A_iB_j occurs if* sampling independence holds on components **A** and **B** for differing decision criteria and if decisional separability holds for both components.

3. If decisional separability fails, then sampling independence is logically unrelated to perceptual independence.

Notice that sampling independence and perceptual independence are logically related only when decisional separability holds. In addition, this theorem does not assume specific distributions and, therefore, holds for the general GRT case. In part 2 of Theorem 8.1, the stipulation that sampling independence holds for differing decision criteria is necessary to ensure that the identification process is not simply due to some idiosyncratic placement of the decision bounds, but that it holds no matter where the (parallel) decision bounds are placed.

The next theorem relates perceptual and decisional separability to perceptual independence (Theorem 4 in Ashby & Townsend, 1986).

Theorem 8.2: Perceptual separability and decisional separability together imply perceptual independence of components **A** and **B** within each stimulus if both of the following conditions hold:

(i) The perceptual representations of all stimuli are Gaussian.
(ii) The subject is responding optimally (in the sense of maximizing probability of correct responses).

The third theorem relates perceptual and decisional separability with marginal response invariance. The first part of this theorem was presented in Ashby and Townsend (1986, Theorem 5) and concerns the general, distribution-free case; the second part was discovered by Kadlec and Townsend (1992) and applies to the Gaussian version of GRT only.

Theorem 8.3: 1. In the general distribution-free case, if perceptual and decisional separabilities hold for components **A** and **B**, then marginal response invariance holds for both components.

2. In the case where all densities are Gaussian and decisional separability holds for components **A** and **B**, perceptual separability holds for component **A** if both of the following conditions are satisfied:

(i) The marginal variances for component **A** are equal across levels of component **B**.
(ii) Marginal response invariance holds for component **A**.

The first part of this theorem indicates that both separabilities together are sufficient for marginal response invariance, but that neither alone is sufficient. Part 2 shows that marginal response invariance provides a useful test of perceptual separability in the Gaussian case under the assumptions that decisional separability holds and that the variances are equal across levels. The latter assumption does not mean that *all* variances need to be equal; in particular, it is not true that the variances of the noise and signal + noise distributions need to be equal. These three theoretical relationships are summarized in Figure 8.3.

Theorems Relating Perceptual and Decisional Separabilities with Signal Detection Parameters

We have recently extended the theoretical relationships from the multidimensional signal detection point of view (Kadlec & Townsend, 1992). The wealth of theoretical results yielded by this approach provides additional observability of perceptual independence and perceptual and decisional separability by the two sets of signal detection parameters. The two sets of parameters—the macroanalytic (marginal) and microanalytic (conditional) d''s and β's—are tied very closely to the GRT representation of the perceptual space in feature-complete factorial designs. Again, these results were shown for the case where there are two components, each with two possible levels, but they readily generalize to more components as well as to more levels. The assumption that each stimulus is represented by a multivariate Gaussian density underlies each of these theorems.

In some of the following theorems, we often need to make assumptions regarding the equality of variances of the marginal densities. However, there are two versions of this assumption. In the weaker case, the assumption of equal marginal variances *across levels of the other component* denotes that the variances of the marginal densities of one component do not change across levels of the other component. However, the variances of the noise and signal + noise densities under consideration need *not* be equal, even for the same component.

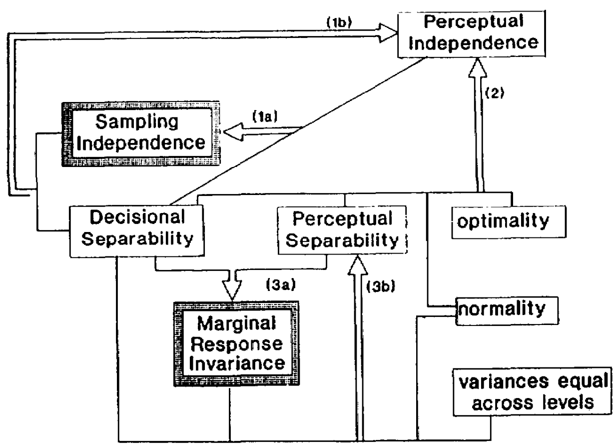

FIG. 8.3. Summary of theoretical relationships in Theorems 8.1 to 8.3 (adapted from Figure 11 in Ashby & Townsend, 1986). Note that these relationships hold in distribution-free cases, except where normality is also required. (Double arrows indicate the direction of an implication, and single lines connect the sufficient conditions. The shaded double boxes indicate observable conditions; conditions in single boxes are unobservable.)

We have already seen this assumption in part 2 of Theorem 8.3. The second, stronger assumption is that *all* marginal variances are equal, across levels of a component as well as the variances of the noise and signal + noise densities, for each component. We will make explicit which assumption is required in each of the theorems.

The next three theorems relate macroanalytic signal detection parameters with perceptual separability. Theorems 8.7 to 8.9 involve perceptual independence and the microanalytic signal detection parameters. For the theorems stated for component A across component B, the same results also hold for component B across component A.

Perceptual Separability, Decisional Separability, and Signal Detection Macroanalytic Parameters: Theorems 8.4–8.6

Theorem 8.4 concerns marginal d''s and perceptual separability.

> *Theorem 8.4:* 1. If component A is perceptually separable from component B, then the marginal d''s for component A will be equal across levels of component B.

> 2. Equal marginal d''s for component A alone do not imply perceptual separability for component A.
>
> 3. Perceptual separability for *both* components holds if and only if the following three conditions hold:
>
> (i) The variances of the marginal densities for each component are equal across levels of the other component.
>
> (ii) The marginal d''s for one component are equal across levels of the other component for both components (i.e., $d'_{\mathbf{A1B}} = d'_{\mathbf{A2B}} = d'_{\mathbf{A}}$ and $d'_{\mathbf{AB1}} = d'_{\mathbf{AB2}} = d'_{\mathbf{B}}$).
>
> (iii) $d'_{\mathbf{AB}} = \sqrt{d'^2_{\mathbf{A}} + d'^2_{\mathbf{B}}}$, where $d'_{\mathbf{AB}}$ is the diagonal d' between the densities f_{11} and f_{22} (we call this the Euclidean diagonal condition).

This theorem is particularly useful for two reasons; it does not depend on decisional separability, and part 3 gives the sufficient as well as the necessary conditions for mutual perceptual separability of two components. This allows us to draw conclusions about the unobservable perceptual separability from a number of testable conditions. Note that conditions (i) and (ii) in part 3 must hold for *both* components. Conditions (ii) and (iii) together indicate that the mean vectors (centers of the densities in Figure 8.1) form a rectangular configuration.

The diagonal d', however, cannot be accurately estimated from feature-complete factorial designs where *all* stimuli are presented to the observer in the same blocks of trials. On any one trial in such an experiment with two components each with two levels, one of four possible stimuli is presented and the observer responds with one of four possible responses. For the diagonal d', the estimate of the probability of a hit is the proportion of trials, when the two-component stimulus is presented, on which both features are correctly reported as present. (Similarly, the probability of a false alarm is the proportion of trials, when the blank stimulus is presented, on which both features are incorrectly reported as present.) However, since four responses are possible, if, on a two-component trial, an observer reports that he or she saw one component, thereby getting a hit on one component but a miss on the other, should this trial be included in the estimate of the probability of a hit? It is not clear whether such trials should be excluded from the estimates, thereby underestimating the probabilities, or included. We have thus suggested (Kadlec & Townsend, 1992) that a slightly different experimental design be used to test this condition. As in the "standard" feature-complete factorial design, in addition to blocks of trials with all stimuli, present separate blocks of trials with only the subset of stimuli necessary to estimate the diagonal d'. The studies discussed later in this chapter have not used this modified design, but this condition should be tested as soon as possible.

The next theorem states the relationships between perceptual and decisional separability and marginal β's:

> *Theorem 8.5:* 1. If decisional separability holds for both components and perceptual separability holds for component A, then the marginal β's for component A will be equal across levels of component B.

2. If decisional separability holds for both components but perceptual separability fails for component A, then it does not logically follow that marginal β's for component *A will be equal.*

3. If decisional separability holds for both components and marginal β's for component A are equal, then it does not necessarily follow that perceptual separability will hold for component A (even when all marginal variances are equal).

This theorem simply states that under the assumption that decisional separability holds for both components, knowing that the marginal β's are equal does not give direct information about perceptual separability. The best we can do when we have support for perceptual separability from the marginal d' results is to obtain additional, but indirect, support for perceptual separability from the equal marginal β's. Conversely, if marginal d''s support perceptual separability (by being equal and satisfying the Euclidean diagonal d' condition), but the marginal β's are not equal, then the contrapositive of part 1 implies that decisional separability must have failed. In the absence of perceptual separability, part 2 states that the marginal β's can be anything, even when decisional separability holds.

Theorem 8.6 sheds more light on decisional separability with the relationships between the marginal response invariance and marginal signal detection parameters.

Theorem 8.6: 1. Marginal d''s for component A will be equal across levels of component **B** if all three of the following conditions are satisfied:

(i) Decisional separability holds for both components.
(ii) Marginal response invariance holds for component A across levels of component **B**.
(iii) The variances of the marginal densities for component A are equal across levels of component **B**.

2. Marginal β's for component A will be equal across levels of component **B** if both of the following conditions are satisfied:

(i) Decisional separability holds for both components.
(ii) Marginal response invariance holds for component A across levels of component **B** (without any assumptions about the marginal variances).

3. Marginal response invariance holds for component **A** if all four of the following conditions are satisfied:

(i) Decisional separability holds for both components.
(ii) Marginal d''s for component A are equal across levels of **B**.
(iii) Marginal β's for component A are equal across levels of **B**.
(iv) *All* marginal variances are equal.

In this theorem, decisional separability is required in each part of the theorem. Also note that under the strongest assumption about the variances (that they are *all* equal) and when decisional separability holds, marginal response invariance holds if and only if marginal d''s and marginal β's are equal. This theorem provides a stronger test of decisional separability, since equality of the variances is also potentially testable. If all the variances are found to be equal, but marginal

response invariance, equal d''s, or equal β's are *not all* either supported or rejected, then decisional separability is logically falsified. If the marginal variances are equal for one component across levels of the other, decisional separability is weakly supported if marginal response invariance holds and marginal d''s are equal (part 1). But even if marginal variances are not equal, some support for decisional separability is obtained when marginal response invariance holds and marginal β's are equal (part 2).

These relationships between the marginal signal detection parameters, marginal response invariance, and the two separabilities are summarized in Figure 8.4.

Perceptual Independence, Decisional Separability and Signal Detection Microanalytic Parameters: Theorems 8.7–8.9

The next set of theorems relates the conditional d''s and β's with perceptual independence. Each theorem assumes that decisional separability holds for both components. These theorems also apply in the feature-complete factorial case,

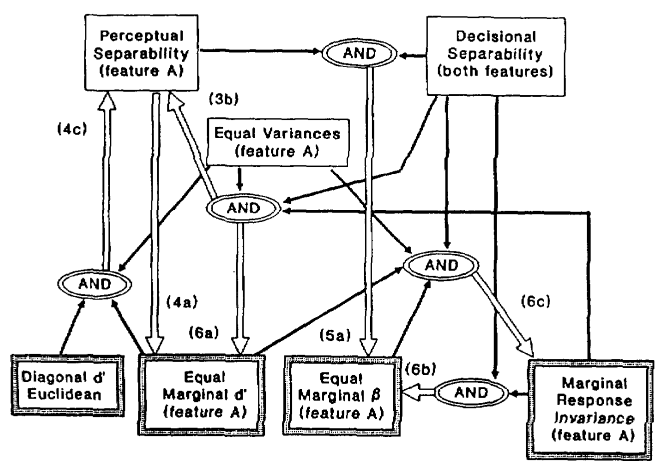

FIG. 8.4. Summary of theoretical relationships in Theorems 8.4 to 8.6 (marginal results). These hold in the Gaussian case. (Double arrows indicate the direction of an implication; solid arrows connect the sufficient conditions. Double-shaded boxes indicate observable conditions; single boxes denote unobservable conditions.)

but since perceptual independence is a within-stimulus property the results pertain to pairs of stimuli. Recall also that the terminology we use is strictly for convenience, and that feature "absence" refers to that portion of the perceptual space below a given criterion and feature "presence" to that above the criterion. For each theorem the results are given for component A conditional on component B; however, analogous results hold for component B conditional on component A by interchanging the roles of A and B in the conditional parameters as well as in the corresponding pairs of stimuli.

Perceptual independence is related with conditional d''s in Theorem 8.7 and with conditional β's in Theorem 8.8. Because these results are quite similar, we first state both, and then discuss them together.

Theorem 8.7: When decisional separability holds for both components, if perceptual independence holds in stimuli A_1B_2 and A_2B_2 (A_1B_1 and A_2B_1), then the d' for component A conditional on the hit (false alarm) of component B (above c_B) will be equal to the d' for component A conditional on the miss (correct rejection) of component B (below c_B), and these will equal the marginal d' for component A at the second (first) level of component B.

Theorem 8.8: When decisional separability holds for both components, if perceptual independence holds in stimuli A_1B_2 and A_2B_2 (A_1B_1 and A_2B_1), then β for component A conditional on the hit (false alarm) of component B will be equal to β for component A conditional on the miss (correct rejection) of component B, and these will equal the marginal β for component A at the second (first) level of component B.

These results are intuitively apparent from the definition of perceptual independence, since if two variables are independent the behavior of one does not influence the other. This fact was one of the original rationales for using the conditional d' and β analyses in the studies by Townsend et al. (1984, 1988). However, the implications in both of these theorems go from the unobservable perceptual independence to the conditional parameters. The converses of these theorems are not true. Thus, as far as support for perceptual independence goes, the best we can hope for by using the microanalyses is to accrue indirect evidence for it. However, data can falsify perceptual independence.

The final theorem relates conditional d''s and β's to sampling independence.

Theorem 8.9: When decisional separability holds for both components:

1. If sampling independence holds in stimuli A_1B_2 and A_2B_2 (A_1B_1 and A_2B_1), then the d' for component A conditional on the hit (false alarm) of component B will be equal to the d' for component A conditional on the miss (correct rejection) on component B.

2. If sampling independence holds in stimuli A_1B_2 and A_2B_2 (A_1B_1 and A_2B_1), then the β for component A conditional on the hit (false alarm) of component B will be equal to the β for component A conditional on the miss (correct rejection) of component B.

Parts 1 and 2 of this result follow directly from Theorems 8.1 and 8.7, and 8.1 and 8.8, respectively. In both parts, the relationships are among two testable conditions under the assumption of decisional separability. This theorem, therefore, provides a clear-cut test of decisional separability; decisional separability will be violated if either (a) sampling independence holds and either conditional d''s or β's fail to be constant, or (b) sampling independence fails and both conditional d''s or β's are constant.

The microanalytic relationships presented in Theorems 8.7 to 8.9 are summarized in Figure 8.5.

Summary and Procedures for Testing Perceptual Separability, Decisional Separability, and Perceptual Independence

The marginal signal detection parameters are related to perceptual and decisional separabilities (Figure 8.4), and the conditional parameters are primarily tied to perceptual independence (Figure 8.5). In Figure 8.4, the implication (4c) corresponding to Theorem 8.4, part 3 is a biconditional implication between perceptual separability for *both* features, equality of the marginal d''s, and variances for *both* features plus the Euclidean diagonal condition. In other words, if marginal d''s and variances are found to be equal for both features and the Euclidean diagonal condition holds, then perceptual separability is logically implied for both features. The importance of this result is that it allows us to draw a strong conclusion about perceptual separability.

These relationships can be used to test for the separabilities and independence of perceptual dimensions. Probabilities for testing sampling independence and marginal response invariance are directly obtainable from confusion matrices from feature-complete factorial designs. Marginal and conditional d''s and β's are also estimable from probabilities contained in confusion matrices. Along with the theoretical relationships presented, these can be used to draw conclusions about perceptual and decisional separability and perceptual independence. The conclusions one can draw from the various combinations of observed marginal and conditional results are shown in truth table format in Tables 8.1 and 8.2, and in flowchart form in Figures 8.6 and 8.7, respectively. Conclusions are presented only for component A; however, by interchanging the roles of A and B, we obtain the same information for component B. We should stress that these tables and figures do not include the test of the Euclidean diagonal d' condition since we cannot test for it in the present data (see discussion following Theorem

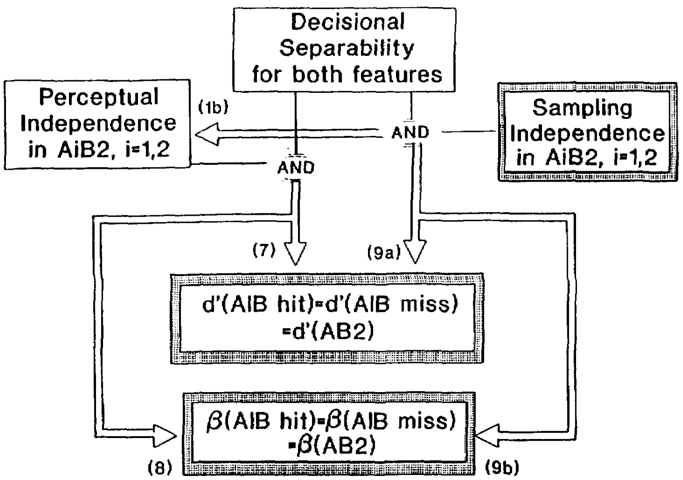

FIG. 8.5. Summary of theoretical relationships in Theorems 8.7 to 8.9 (conditional results). These hold in the Gaussian case. (Double arrows indicate the direction of an implication; solid arrows connect the sufficient conditions. Double-shaded boxes indicate observable conditions; single boxes denote unobservable conditions.)

8.4).[2] The conclusions would change only for perceptual separability when the marginal d''s were equal for both components but the Euclidean diagonal condition did not hold.

In all but one theoretical relationship (Theorem 8.4 part 3), the logical implications go from the unobservable (perceptual separability or perceptual independence) to the observable signal detection parameters. Positive empirical verification of the theoretical concepts thus cannot be directly obtained; however, data may be used in the contrapositive form of the implications to logically imply their failure. For example, equal marginal d''s for component **A** across levels of component **B** provide indirect support for perceptual separability in the sense that the data do not falsify it. However, *unequal* marginal d''s for component **A** across levels of **B** logically imply that perceptual separability has failed. The

[2]In the following section where we illustrate the method, we assume the Euclidean diagonal d' condition is also satisfied whenever we obtain evidence for equal marginal d''s for *both* components, and thereby that the means form a rectangular configuration. We only assume this for illustrative purposes, and we stress that this assumption should be tested as soon as possible. Following such a test, some conclusions we draw in the examples in the following section may change.

TABLE 8.1
Truth Table for Combinations of Observed Marginal Results and the Corresponding Conclusions

Observed Results			*Conclusions*		
Marginal Response Invariance for A ?	*Marginal d'_A Equal ?*	*Marginal β_A Equal ?*	*Decisional Separability for A and B*	*Perceptual Separability for A*	*Equal Marginal Variances for A*
T	T	T	w-support (6b)	w-support (4a, 5a)[a]	w-support (6a)
T	T	F	s-failed for one or both (5a, 6b)	w-support (4a)	?
T	F	T	w-support (6b)	s-failed (4a)	s-failed (6a, 3b)
T	F	F	s-failed for one or both (6b)	s-failed (4a)	?
[b]F	T	T	s-failed[b] (6c)	w-support (4a, 5a)	s-failed[b] (6c)
F	T	F	s-failed for one or both (5a)	w-support (4a)	?
F	F	T	?	s-failed (4a)	?
F	F	F	?	s-failed (4a)	?

[a]If the same pattern of results is also found for **B** and the Euclidean diagonal d' condition is supported, then perceptual separability is s-supported for **A** and **B**.
[b]In this case, either decisional separability has failed or the marginal variances are not equal, or both. This will be denoted by ?(no).

former weak sense of empirical support will be indicated as w-support to distinguish it from the stronger s-support (or s-failure) resulting from a logical implication.

ILLUSTRATION OF THE TESTS FOR SEPARABILITIES AND PERCEPTUAL INDEPENDENCE

The testing procedure discussed and summarized in Figures 8.6 and 8.7 and Tables 8.1 and 8.2 is illustrated in this section. A directly applicable example (two components with two levels) is presented first. This will be followed by analyses to illustrate the two ways of generalizing to more than two dimensions and to more than two levels on each of two components.

TABLE 8.2
Truth Table for Combinations of Observed Conditional Results and the Corresponding Conclusions

Observed Results				Conclusions		
d' for A *Conditional on*	*β for* A *Conditional on*	*Sampling Independence in Stimuli*		*Decisional Separability for* A *and* B	*Perceptual Independence in Stimuli*	
B *hit* = B *miss* (B *cr* = B *fa*)		A_1B_2 (A_1B_1)	A_2B_2 (A_2B_1)		A_1B_2 (A_1B_1)	A_2B_2 (A_2B_1)
T	T	T or F	T or F	w-support (7 & 8)	w-support (7 & 8)	
T	F	T	T	s-failed	?	?
F	T	T	T		(possibly w-supported)	
F	F	T	T			
T	F	F	F	Either DS failed or PI failed in at least one stimulus, or both. If DS supported (in marginal results), then PI s-failed in at least one of these stimuli.		
F	T	F	F			
F	F	F	F			

Two-Dimensional Case with Two Levels on each Dimension

An experiment by Townsend, Hu, and Ashby (1981; abbreviated THA 1981) used a feature-complete factorial design with a vertical line (|) and a horizontal line (—), each either present or absent (see Table 8.3). There were two experimental conditions in this study, a gap and a connected condition, but we will consider data only from the connected condition.

Some of the estimates of marginal response invariance and sampling independence probabilities and marginal and conditional d''s and β's for each of the two components across levels of the other component were reported previously (Ashby & Townsend, 1986; Kadlec & Townsend, 1992; Townsend et al., 1981) and will not be presented here (but are available from the authors upon request). The conclusions of these tests are shown in Table 8.4. Because of the variability in the patterns of results for the individual observers, we will describe the conclusions in detail only for observer 1, and leave the others to the reader to verify.

For observer 1, marginal response invariance held for each component across levels of the other component. In addition, marginal d' and β estimates were found to be equal for both components across levels of the other component. The path leading to the top of Figure 8.6 (or the top row of Table 8.1) thus applies for

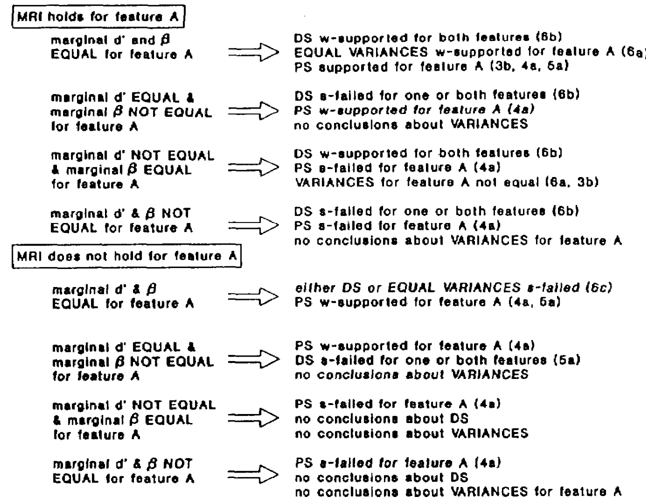

FIG. 8.6. Flowchart for testing perceptual and decisional separabilities and the equal variance assumption from observed marginal results. See also Table 8.1 for an alternative presentation.

both features, indicating w-support for perceptual separability[3] and w-support fo decisional separability.

From the conditional results, however, we find that, although sampling inde pendence held in each stimulus, for each pair of stimuli either the conditional d''s or the conditional β's or both failed to be equivalent. The lower path in Figur 8.7 (alternatively the second set of rows in Table 8.2) should be consulted in thi case, and we conclude that decisional separability has s-failed. This is a logica consequence and thus overrides our previous w-support of decisional sepa rability. Our overall conclusion is then that decisional separability has s-failed fo this observer, and no strong conclusions can be drawn about perceptual indepen dence. However, the conclusion that perceptual separability holds for both com ponents across levels of the other component remains viable.

This data set was also analyzed by Ashby and his colleagues in previou reports. Ashby and Townsend (1986) applied their test of the separabilities an

[3]We remind the reader that if we had a test of the Euclidean diagonal d' condition for these dat and it was empirically supported, then perceptual separability would be logically implied and th s-supported.

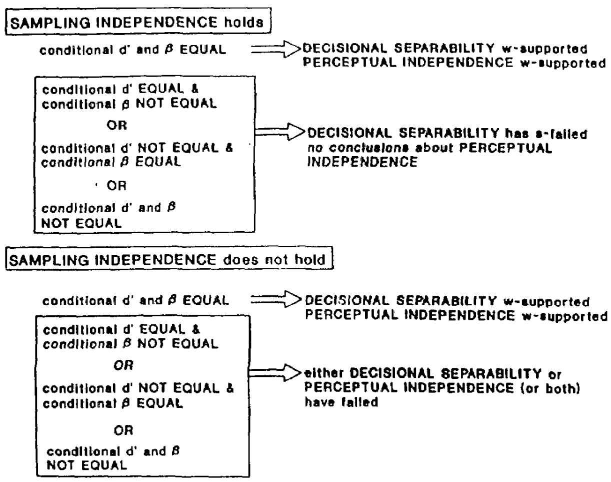

FIG. 8.7. Flowchart for testing perceptual independence and decisional separability from observed conditional results. See also Table 8.2 for an alternative presentation.

perceptual independence (based on Theorems 8.1 and 8.3, which relate sampling independence and marginal response invariance with the separabilities; see Figure 12 in Ashby & Townsend, 1986, p. 173), and concluded that the null hypothesis of perceptual and decisional separabilities could not be rejected for any of the observers in this study. Because sampling independence was tested only on A_2B_2 trials (see Townsend et al., 1981), Ashby and Townsend tested marginal response invariance for each component only at the second level and concluded

TABLE 8.3
Stimulus Components and Their Levels in the Three Studies Discussed

Study	*Stimulus Components*	*Levels of Components*
THA 1981	— (A)	0, 1 (absent, present)
	— (B)	0, 1
Nosofsky 1986	Angle of radial line (A)	50°, 53°, 56°, 59°
	Size of semicircle (B)	.478, .500, .522, .544 cm
THE 1984	| (A)	0, 1 (absent, present)
	— (B)	0, 1
	\ (C)	0, 1
	⊂ (D)	0, 1

TABLE 8.4
Summary of Results and Conclusions for THA 1981 Data

	O_1	O_2	O_3	O_4
MACROANALYSES				
Testing A(—) across B(|)				
Marginal response invariance?	Yes	No	Yes	No
Marginal *d'* equal?	Yes	No	Yes	Yes
Marginal β equal?	Yes	Yes	Yes	Yes
Testing B(|) across A(—)				
Marginal response invariance?	Yes	Yes	Yes	No
Marginal *d'* equal?	Yes	Yes	Yes	Yes
Marginal β equal?	Yes	Yes	No	No
Perceptual separability for A	Yes	No	w-yes	w-yes
Equal variances for A	w-yes	?	w-yes	? (no)
Perceptual separability for B	Yes	w-yes	w-yes	w-yes
Equal variances for B	w-yes	w-yes	?	?
Decisional separability for both (from marginal results)	w-yes	w-yes	No	No
MICROANALYSES				
Sampling independence holds in				
A1B1 ( )?	Yes	Yes	Yes	Yes
A1B2 (|)?	Yes	Yes	Yes	Yes
A2B1 (—)?	Yes	Yes	Yes	Yes
A2B2 (□)?	Yes	Yes	Yes	Yes
Conditional *d'*s equal for				
— given | hit vs. miss?	Yes	No	Yes	Yes
— given | cr vs. fa?	Yes	No	No	No
| given — hit vs. miss?	No	No	Yes	No
| given — cr vs. fa?	No	No	No	No
Conditional β's equal for				
— given | hit vs. miss?	No	Yes	Yes	Yes
— given | cr vs. fa?	No	Yes	No	No
| given — hit vs. miss?	No	Yes	Yes	No
| given — cr vs. fa?	No	No	No	No
Decisional separability (overall conclusions)	No	No	No	No
Perceptual independence				
in A_1B_2 and A_2B_2	?	?	w-yes	w-yes
in A_1B_1 and A_2B_1	?	?	?	?
in A_2B_1 and A_2B_2	?	?	w-yes	?
in A_1B_1 and A_1B_2	?	?	?	?

that marginal response invariance held for each observer. Here we tested sampling independence in each of the four stimuli, as well as marginal response invariance for each component at both levels, and found that marginal response invariance failed at the first level for observers 2 and 4. Our conclusions from the macroanalyses are thus slightly different, particularly with respect to decisional separability.

Ashby and Perrin (1988, experiment 3) also used this data set to fit two versions of the Gaussian general recognition model. Both models assumed perceptual independence, decisional separability, and constant density means for each component across levels of the other component, but differed in the assumptions made for the variances. In the six-parameter model, they assumed perceptual separability by estimating only the variances for the vertical and horizontal components, regardless of the level of the other component. In the 10-parameter model, however, the variances for the vertical and horizontal components were allowed to vary for each component across levels of the other component, thus allowing a weak violation of perceptual separability. For the observers in the connected condition, Ashby and Perrin found that the 10-parameter model significantly improved the fit over the 6-parameter model only for observer 4; but both models were still rejected for this observer. Both models were also rejected for observer 1, but both fit observers 2 and 3. Consistently with these results, our analyses also show that observer 4 was the only one that violated the equal-variance assumption. Our conclusions indicate that, although perceptual separability was supported for both components for observers 1 and 4, these two observers obtained the largest number of unequal conditional d' and β estimates, indicating failure of perceptual independence in at least some of the stimuli. In addition, we found that decisional separability has failed for each observer. Although the relative importance of decisional separability and perceptual independence is unknown, it appears that failure of perceptual independence may have contributed to the poorer fit of the models of Ashby and Perrin.

Two-Dimensional Case with Four Levels on each Dimension

The generalization of the analyses to a situation with more than two levels on each dimension will be demonstrated with data from Nosofsky (1986). Nosofsky used a feature-complete factorial design of stimuli with two components, a semicircle and a radial line, each at one of four possible levels (see Table 8.3). Because of the large amount of data, we will discuss only one of the two observers in this study. Table 8.5 gives the estimates for the marginal response invariance probabilities and marginal d''s and β's for observer 1.

The relevant comparisons are across the four rows within each column. Marginal response invariance holds for angle of radial line (component **A**) across semicircle sizes (**B**) at all levels of angle except the largest; marginal d''s are constant, but marginal β's are not equal. This implies w-support for perceptual separability and s-failure of decisional separability for angle across semicircle sizes. Marginal response invariance fails for all levels of semicircle sizes (**B**) across angle of radial line (**A**), marginal d''s remain constant, but marginal β's are not equal, a pattern of results also indicating w-support for perceptual separability and s-failure for decisional separability for component **B** across levels of

TABLE 8.5
Macroanalyses for Observer 1 in Nosofsky (1986)

Dimension **A** *(Angle of Radial Line) Across* **B** *(Size of Semicircle)*

	A_1B_j		A_2B_j		A_3B_j		A_4B_j
Marginal response invariance[a]							
j = 1 (.478 cm)	.761		.581		.634		.630*
j = 2 (.500 cm)	.796		.518		.609		.684
j = 3 (.522 cm)	.800		.561		.560		.701
j = 4 (.544 cm)	.764		.592		.593		.730*
Marginal d' estimates[b]							
j = 1 (.478 cm)		1.68		1.83		1.63	
j = 2 (.500 cm)		1.46		1.63		1.62	
j = 3 (.522 cm)		1.59		1.60		1.54	
j = 4 (.544 cm)		1.59		1.57		1.58	
Marginal β estimates[c]							
j = 1 (.478 cm)		0.35#		0.63#		1.27#	
j = 2 (.500 cm)		0.59		0.87		1.72	
j = 3 (.522 cm)		0.63		1.22		1.94	
j = 4 (.544 cm)		0.83#		1.37		2.46#	

Dimension **B** *(Size of Semicircle) Across* **A** *(Angle of Radial Line)*

	A_iB_1		A_iB_2		A_iB_3		A_iB_4
Marginal response invariance[a]							
i = 1 (50°)	.583		.775		.709		.748
i = 2 (53°)	.643		.628		.689		.682
i = 3 (56°)	.693		.682		.635		.632
i = 4 (59°)	.739		.650		.631		.588
Marginal d' estimates[b]							
i = 1 (50°)		1.65		1.76		1.09	
i = 2 (53°)		1.77		1.65		1.11	
i = 3 (56°)		1.80		1.73		1.04	
i = 4 (59°)		1.80		1.79		1.18	
Marginal β estimates[c]							
i = 1 (50°)		0.83		0.72#		1.26#	
i = 2 (53°)		0.91		0.59		1.09	
i = 3 (56°)		0.90		0.65		0.99	
i = 4 (59°)		0.73		0.61		0.97	

Notes: [a]* indicates a significant difference.

[b]* indicates a d' greater than 2 standard deviations from the column mean, using Grier's (1971) nonparametric test.

[c]# indicates that the B index (Grier, 1971) of the β varied by more than 10% from the other β's.

A. No conclusions about the marginal variances can be drawn for either dimension.

Regarding the perceptual versus decisional effects on identification performance, Nosofsky (1985b, 1986) reached similar conclusions for this observer by fitting various Multidimensional Scaling–similarity choice models. The best Multidimensional Scaling solution showed the 16 stimuli arranged in a regular rectagular configuration. In a perceptual space this would correspond to perceptual separability of both components, if equal variances are assumed. In addition, Nosofsky found that this observer's decisions on one component were *not* independent of the level of the other component. In particular, Nosofsky (1985b, p. 426) reports that this observer was biased against making identical responses for both components in a given stimulus [e.g., (3,3)]. This is also consistent with our conclusion that decisional separability has failed for this observer.

Sampling independence probabilities are given in Table 8.6, and the conditional d' and β estimates for each pair of stimuli are shown in Table 8.7. Sampling independence was found in all stimuli; thus, the top two rows of Table 8.2 or the top path in Figure 8.7 apply. The conclusions regarding perceptual independence and decisional separability from these conditional results are given in the last two columns of Table 8.7. From these results we again conclude that overall decisional separability has failed for both dimensions. For perceptual independence, since the test is for pairs of stimuli, for a given stimulus we must compare the results obtained from each test that involves that stimulus. For example, stimulus A_1B_2 appears in the first two rows under the heading dimension A as well as in the fourth row under dimension B. Since the test for A_1B_2 and A_1B_3, however, indicated that perceptual independence was w-supported for both of these stimuli, the result for A_1B_2 is that perceptual independence is w-supported. Perceptual independence in A_1B_1 may remain in question, however, since the only other test involving A_1B_1 is also questionable w-support. From these analyses we conclude that only the stimuli A_1B_1, A_2B_2, and A_4B_3

TABLE 8.6
Sampling Independence for Observer 1 in Nosofsky (1986) Data

Stimulus	*P(x,y)*	*P(x)P(y)*	*Stimulus*	*P(x,y)*	*P(x)P(y)*
A_1B_1	.424	.444	A_1B_3	.572	.567
A_2B_1	.402	.376	A_2B_3	.403	.387
A_3B_1	.436	.439	A_3B_3	.351	.356
A_4B_1	.488	.466	A_4B_3	.420	.442
A_1B_2	.545	.538	A_1B_4	.561	.571
A_2B_2	.332	.325	A_2B_4	.426	.404
A_3B_2	.411	.415	A_3B_4	.400	.375
A_4B_2	.450	.445	A_4B_4	.416	.429

Note: None of the probabilities were significantly different on a two-tailed *Z* test at α = .25 level.

TABLE 8.7
Conditional d' and β Estimates for Perceptual Independence for Observer 1 in Nosofsky (1986)

Stimuli	*d' (>c)*[a]		*d' (<c)*	*β (>c)*		*β (<c)*	*PI?*[b]	*DS?*
Dimension **A** *(angle of radial line), conditional on* **B** *(semicircle size)*								
A_1B_1 and A_1B_2	1.60		1.71	0.56	#	0.29	?(w-yes)	s-no
A_1B_2 and A_1B_3	1.84		1.80	0.61		0.72	w-yes	w-yes
A_1B_3 and A_1B_4	1.59		1.74	1.30		1.11	w-yes	w-yes
A_2B_1 and A_2B_2	1.44	*	2.09	0.69	#	0.11	?(w-yes)	s-no
A_2B_2 and A_2B_3	1.63		1.63	0.93	#	0.63	?(w-yes)	s-no
A_2B_3 and A_2B_4	1.68		1.31	1.73		1.68	w-yes	w-yes
A_3B_1 and A_3B_2	1.56		1.77	0.65	#	0.51	?(w-yes)	s-no
A_3B_2 and A_3B_3	1.59		1.73	1.30	#	0.74	?(w-yes)	s-no
A_3B_3 and A_3B_4	1.51		1.78	1.99		1.68	w-yes	w-yes
A_4B_1 and A_4B_2	1.56		1.53	0.95	#	0.63	?(w-yes)	s-no
A_4B_2 and A_4B_3	1.56		1.66	1.67	#	0.88	?(w-yes)	s-no
A_4B_3 and A_4B_4	1.61		1.56	2.74	#	1.96	?(w-yes)	s-no
Dimension **B** *(semicircle size), conditional on* **A** *(angle of radial line)*								
A_1B_1 and A_2B_1	1.72		1.46	0.64	#	1.08	?(w-yes)	s-no
A_2B_1 and A_3B_1	1.75		1.76	0.72		0.70	w-yes	w-yes
A_3B_1 and A_4B_1	1.16		0.90	1.22		1.36	w-yes	w-yes
A_1B_2 and A_2B_2	1.71	*	2.37	1.03	#	0.14	?(w-yes)	s-no
A_2B_2 and A_3B_2	1.70	*	1.41	0.63	#	0.42	?(w-yes)	s-no
A_3B_2 and A_4B_2	1.07		1.27	1.12		0.91	w-yes	w-yes
A_1B_3 and A_2B_3	1.77		1.87	0.96	#	0.67	?(w-yes)	s-no
A_2B_3 and A_3B_3	1.74		1.66	0.76	#	0.38	?(w-yes)	s-no
A_3B_3 and A_4B_3	1.04		1.05	1.12	#	0.70	?(w-yes)	s-no
A_1B_4 and A_2B_4	1.82		1.79	0.65	#	0.92	?(w-yes)	s-no
A_2B_4 and A_3B_4	1.76		1.85	0.69	#	0.46	?(w-yes)	s-no
A_3B_4 and A_4B_4	1.29		1.00	1.14		1.28	w-yes	w-yes

Notes: See notes to Table 8.5 for significance criteria.

[a]The notation d' (>c) means d' conditional above the criterion between the given pairs of stimuli, and d' (<c) stands for d' conditional below the criterion. Similar notation is used for the conditional β's.

[b]PI stands for perceptual independence, DS for decisional separability.

have at best uncertain w-support; for all other stimuli at least one of the tests in which they appeared indicated w-support for perceptual independence.

Four-Dimensional Case with Two Levels on each Dimension

Townsend, Hu, and Evans (1984; abbreviated THE 1984) used stimuli similar to those of Townsend et al. (1981), but with four components each either present or absent: a vertical line (|), horizontal line (—), a diagonal line (\), and a curved line (⊂) (see Table 8.3). This data set will illustrate how the analyses can be generalized to stimuli with a four-dimensional representation. Marginal response invariance probabilities, marginal d''s and marginal β's, as well as the conclusions about perceptual and decisional separabilities and marginal variances are presented in Table 8.8 for one of the four observers in this study. (Again because of the large volume of results, only those for observer 3 are shown.)

The results are given for each component across levels of one other component, at fixed levels of the third and fourth components (see discussion at end of the third section for our rationale). Conclusions for the separabilities and marginal variances reached by these analyses are given in the last three columns of Table 8.8. Perceptual separability was w-supported in most cases, failing in only three cases all of which involved component A (vertical line). Decisional separability s-failed in all but four tests; in three of the four tests where decisional separability was w-supported, the stimulus contained only two components. Finally, no conclusions could be drawn about the equality of the marginal variances in many cases.

Probabilities for testing sampling independence are given in Table 8.9. Begin-

TABLE 8.8
Observed Marginal Estimates and Summary of Separability Results for Observer 3 THE 84

	MRI		*Marginal*		*Marginal*				*Marginal* σ^2
Component	P_1	P_2	d'_1	d'_2	β_1	β_2	*PS?*[a]	*DS?*	*Equal?*
A_2 across B									
at C_1D_1	.636	.560*	2.29	1.89	3.82	2.41*	w-yes	no	?
at C_1D_2	.577	.483*	1.61	1.35	1.09	1.23	w-yes	?(no)	?(no)
at C_2D_1	.524	.527	2.15	1.85	3.37	1.76*	w-yes	no	?
at C_2D_2	.404	.446*	1.20	2.36*	1.35	3.66*	no	?	?
A_2 across C									
at B_1D_1	.623	.584*	2.29	2.15	3.82	3.37	w-yes	? (no)	? (no)
at B_1D_2	.614	.477*	1.61	1.20	1.09	1.35*	w-yes	no	?
at B_2D_1	.517	.547	1.89	1.85	2.41	1.76*	w-yes	no	?
at B_2D_2	.456	.500*	1.35	2.36*	1.23	3.66*	no	?	?
A_2 across D									
at B_1C_1	.606	.604	2.29	1.61	3.82	1.09*	w-yes	no	?
at B_1C_2	.500	.427*	2.15	1.20*	3.37	1.35*	no	?	?
at B_2C_1	.500	.436*	1.89	1.35	2.41	1.23*	w-yes	no	?
at B_2C_2	.467	.416*	1.85	2.36	1.76	3.66*	w-yes	no	?
B_2 across A									
at C_1D_1	.463	.594*	1.88	2.17	2.56	2.51	w-yes	? (no)	? (no)
at C_1D_2	.557	.500*	2.10	1.83	1.73	2.02	w-yes	? (no)	? (no)
at C_2D_1	.526	.527	2.01	1.83	2.82	1.68*	w-yes	no	?
at C_2D_2	.404	.466*	1.56	2.10	1.26	2.27*	w-yes	no	?

(*continued*)

TABLE 8.8 (*Continued*)

	MRI		*Marginal*		*Marginal*				*Marginal* σ^2
Component	P_1	P_2	d'_1	d'_2	β_1	β_2	*PS?*[a]	*DS?*	*Equal?*
B_2 across C									
at A_1D_1	.566	.593	1.88	2.01	2.56	2.82	w-yes	w-yes	w-yes
at A_1D_2	.503	.514	2.10	1.56	1.73	1.26*	w-yes	no	?
at A_2D_1	.517	.547	2.17	1.83	2.51	1.68*	w-yes	no	?
at A_2D_2	.456	.500*	1.83	2.10	2.02	2.27	w-yes	? (no)	? (no)
B_2 across D									
at A_1C_1	.466	.570*	1.88	2.10	2.56	1.73*	w-yes	no	?
at A_1C_2	.496	.480	2.01	1.56	2.82	1.26*	w-yes	no	?
at A_2C_1	.500	.436*	2.17	1.83	2.51	2.02	w-yes	? (no)	? (no)
at A_2C_2	.467	.416*	1.83	2.10	1.68	2.27	w-yes	? (no)	? (no)
C_2 across A									
at B_1D_1	.697	.590*	2.02	2.03	1.76	2.47*	w-yes	no	?
at B_1D_2	.480	.514	1.70	1.57	1.69	1.20*	w-yes	no	?
at B_2D_1	.526	.527	1.62	1.80	1.28	1.51*	w-yes	no	?
at B_2D_2	.404	.466*	1.67	1.79	1.84	2.03	w-yes	? (no)	? (no)
C_2 across B									
at A_1D_1	.713	.660*	2.02	1.62	1.76	1.28*	w-yes	no	?
at A_1D_2	.497	.504	1.70	1.67	1.69	1.84	w-yes	w-yes	w-yes
at A_2D_1	.524	.527	2.03	1.80	2.47	1.51*	w-yes	no	?
at A_2D_2	.467	.446	1.57	1.79	1.20	2.03*	w-yes	no	?
C_2 across D									
at A_1B_1	.703	.493*	2.02	1.70	1.76	1.69	w-yes	? (no)	? (no)
at A_1B_2	.496	.480	1.62	1.67	1.28	1.84*	w-yes	no	?
at A_2B_1	.500	.427*	2.03	1.57	2.47	1.20*	w-yes	no	?
at A_2B_2	.467	.416*	1.80	1.79	1.51	2.03*	w-yes	no	?
D_2 across A									
at B_1C_1	.590	.590	2.02	2.24	3.04	2.51	w-yes	w-yes	w-yes
at B_1C_2	.480	.514	2.12	2.00	2.44	1.79*	w-yes	no	?
at B_2C_1	.557	.500*	1.89	2.02	2.03	1.66*	w-yes	no	?
at B_2C_2	.404	.466*	2.39	2.41	3.26	2.35*	w-yes	no	?
D_2 across B									
at A_1C_1	.540	.567	2.02	1.89	3.04	2.03*	w-yes	no	?
at A_1C_2	.497	.404*	2.12	2.39	2.44	3.26*	w-yes	no	?
at A_2C_1	.577	.483*	2.24	2.02	2.51	1.66*	w-yes	no	?
at A_2C_2	.467	.446	2.00	2.41	1.79	2.35*	w-yes	no	?
D_2 across C									
at A_1B_1	.527	.507	2.02	2.12	3.04	2.44	w-yes	w-yes	w-yes
at A_1B_2	.503	.514	1.89	2.39	2.03	3.26	w-yes	no	?
at A_2B_1	.614	.477*	2.24	2.00	2.51	1.79*	w-yes	no	?
at A_2B_2	.456	.500*	2.02	2.41	1.66	2.35*	w-yes	no	?

Notes: See notes to Table 8.5 for significance criteria.

[a]PS stands for perceptual separability, DS stands for decisional separability, and MRI for marginal response invariance.

TABLE 8.9
Sampling Independence for Observer 3 in THE 84 Study

4-Way Sampling Independence

Stimulus	*P(x,y,z,w)*	*P(x)P(y)P(z)P(w)*
$A_2B_2C_2D_2$	.373	.304*

3-Way Sampling Independence

Stimulus	*P(x,y,z)*	*P(x)P(y)P(z)*
$A_2B_2C_2$ at D_1	.467	.406*
at D_2	.416	.379*
$A_2B_2D_2$ at C_1	.456	.381*
at C_2	.500	.440*
$A_2C_2D_2$ at B_1	.467	.364*
at B_2	.446	.408*
$B_2C_2D_2$ at A_1	.404	.380
at A_2	.466	.414*

2-Way Sampling Independence

Stimulus	*P(x,y)*	*P(x)P(y)*	*Stimulus*	*P(x,y)*	*P(x)P(y)*
A_2B_2 at C_1D_1	.550	.512*	B_2C_2 at A_1D_1	.549	.514*
at C_1D_2	.519	.491	at A_1D_2	.507	.502
at C_2D_1	.597	.541*	at A_2D_1	.577	.553
at C_2D_2	.566	.548	at A_2D_2	.556	.516*
A_2C_2 at B_1D_1	.557	.498*	B_2D_2 at A_1C_1	.593	.564
at B_1D_2	.507	.479	at A_1C_2	.564	.559
at B_2D_1	.567	.551	at A_2C_1	.596	.545*
at B_2D_2	.502	.509	at A_2C_2	.616	.598
A_2D_2 at B_1C_1	.671	.587*	C_2D_2 at A_1B_1	.570	.522*
at B_1C_2	.574	.485*	at A_1B_2	.548	.516
at B_2C_1	.603	.543*	at A_2B_1	.617	.571*
at B_2C_2	.643	.589*	at A_2B_2	.582	.555

Note: * indicates a significant difference at the α = .25 level (two-tailed Z test).

ning with the mutual sampling independence of all four components (top of Table 8.9), we see that it has failed. Three-way sampling independence, however, was found to hold for components **B** (horizontal), **C** (diagonal), and **D** (curve) at the first level of **A** (when the vertical feature was absent). This must imply pairwise independence of these three components at the first level of **A** (**B** and **C** at A_1D_2, **B** and **D** at A_1C_2, and **C** and **D** at A_1B_2), and, in fact, we find that this is so. For all other three-way combinations of the components, sampling independence has failed, and examination of the pairwise tests will indicate where the dependencies occurred.

Conditional signal detection estimates for testing pairwise perceptual independence of components **A** and **B**, at *all* possible combinations of levels of **C** and **D**, are presented in Table 8.10 (refer to Table 8.2 or Figure 8.7 for the conclusions). Similar analyses would be done for each pair of components, but because of numerous possible combinations (see Table 8.9), we feel it is more instructive to show fewer results with greater clarity.

As with the two-dimensional example with four levels, we test pairs of stimuli at a time. Perceptual independence for a given stimulus can only be concluded when none of the tests that involve that stimulus show failure of perceptual independence that cannot be attributed to another stimulus.

SUMMARY AND FUTURE DIRECTIONS

Human perception of visual patterns has often been studied from a "dimensional" perspective, whereby a stimulus is assumed to be decomposed by the perceptual system into a number of individual components. By varying the composition of the physical stimuli, we can investigate various influences of one component on the other. In this chapter, we have employed the framework of GRT to define and interrelate five notions of "perceptual independence" that have appeared in the literature to connote that one component does not influence another. We have given theoretical results that relate multidimensional Signal Detection Theory to the various notions of "independence" within GRT and that allow us to test these various "independencies." The method based on these theoretical results was illustrated with three examples from the literature.

TABLE 8.10
Perceptual Independence Tests for Components A and B Using Estimates of Conditional d' and β for Observer 3 in THE 1984

Components C and D	C_1D_1	C_1D_2	C_2D_1	C_2D_2
Sampling Independence of A_2 and B_2?	No	Yes	No	Yes
d' $(A_2\|B_2)$	1.94	1.43	2.07	1.80
d' $(A_2\|B_1)$	1.72	1.28	1.35*	1.40
β $(A_2\|B_2)$	1.93	1.11	1.38	1.43
β $(A_2\|B_1)$	3.76#	1.63#	2.33#	1.46
Perceptual independence in A_2B_2 and A_1B_2	? (w-yes)	? (w-yes)	? (w-yes)	w-yes
d' $(B_2\|A_2)$	2.26	2.04	2.17	1.66
d' $(B_2\|A_1)$	2.13	1.31*	1.11*	1.30
β $(B_2\|A_2)$	1.88	2.13	1.53	1.16
β $(B_2\|A_1)$	4.89#	1.66#	1.71	1.26
Perceptual independence in A_2B_2 in A_2B_1	? (no)	? (w-yes)	? (no)	w-yes
Decisional separability	? (no)	s-no	? (no)	w-yes

See notes to previous tables for significance criteria.

At the present time, these results have been employed only in feature-complete factorial designs with accuracy measures as the dependent variable. But the method is quite new, and there are extensions that could further strengthen it or render it more broadly applicable.

Many of the theorems for the marginal results assume that the marginal (or all) variances are equal. An experimental design that could simultaneously test this assumption along with the tests of the separabilities would therefore be desirable. There are two possible ways such a test may be incorporated, both of which involve the construction of a receiver operating characteristic (ROC) curve. This curve simply plots the probability of false alarms as a function of the probability of hits. From an isosensitivity curve, which results when all the d''s are equivalent (a testable condition that we already employ), an estimate of the "signal" variance can be obtained. The feature-complete factorial designs we discussed here provide only one point in the ROC space, which is insufficient to obtain an isosensitivity curve *and* an estimate of the "signal" variance. The two ways of obtaining more points for the ROC curve are by using confidence ratings rather than straight identification of each component-level, or awarding differential payoffs for different components in the stimulus set. Along with a test of the Euclidean diagonal condition (see discussion following Theorem 8.4), a strong test of perceptual separability would be achieved. Confidence ratings have been used by other investigators in developing and using methods somewhat similar to ours for testing "independence" of components (Ashby, 1988; Wickens & Olzak, 1989).

An interesting and direct application of the method presented here is to the kind of feature-complete factorial designs that Melara and Marks (1990a) used in their studies on uncovering "primary" perceptual dimensions. This involves different orientations of axes of the physical dimensions from which the stimuli are constructed. Although Melara and Marks used response times, the same stimuli could be used with accuracy measures. Presumably, there will be a most preferred orientation where perceptual and decisional separabilities will hold but fail at other orientations. As for perceptual independence of dimensions in a given stimulus, if the variances are equal in both dimensions and perceptual independence holds in one orientation, then perceptual independence should hold at all orientations. These analyses could provide interesting results for identifying "primary" perceptual dimensions in the visual domain, as well as other sensory arenas.

APPENDIX

The first part of this appendix defines marginal d' and β parameters of signal detection macroanalyses; in the second part, conditional d' and β of signal detection microanalyses are defined.

Signal Detection Macroanalyses

The d' and β parameters of signal detection macroanalysis are based on the marginal densities (see Figure 8.1). Theoretically these are obtained by integrating the joint densities in the perceptual space over the variable *not* of interest. These marginal densities are then used as the "noise" (that density where the feature was absent from the stimulus) and signal + noise (where the feature was present in the stimulus) densities, as in Signal Detectability Theory of Green and Swets (1974), to obtain the d' and β parameters. The theoretical formulas that follow correspond to the methods of estimation of these parameters used by Townsend et al. (1984, 1988).

We denote the mean vector and covariance matrix of density $f_{ij}(x, y)$ by

$$\boldsymbol{\mu}_{ij} = \begin{bmatrix} \mu_{Xij} \\ \mu_{Yij} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma}_{ij} = \begin{bmatrix} \sigma^2_{Xij} & \sigma_{XYij} \\ \sigma_{XYij} & \sigma^2_{Yij} \end{bmatrix},$$

respectively. Note that for a normal density, the covariance term is $\sigma_{XYij} = \rho_{ij}\sigma_{Xij}\sigma_{Yij}$, where ρ_{ij} is the correlation coefficient between X and Y in density f_{ij}. The marginal d''s for feature **A** at the j-th level of feature **B** can then be written as

$$d'_{\mathbf{A}\mathbf{B}j} = \frac{\mu_{X2j} - \mu_{X1j}}{[(\sigma^2_{X2j} + \sigma^2_{X1j})/2]^{1/2}}, \qquad j = 1,2.$$

Similarly,

$$d'_{\mathbf{A}i\mathbf{B}} = \frac{\mu_{Yi2} - \mu_{Yi1}}{[(\sigma^2_{Yi2} + \sigma^2_{Yi1})/2]^{1/2}}, \qquad i = 1,2.$$

are the marginal d''s for the detection of feature **B** at the i-th level of feature **A**.

As in Signal Detectability Theory, the marginal β's are defined as the likelihood ratios of the marginal densities, evaluated at the corresponding criterion values, $c_{\mathbf{A}}$ (criterion for presence of feature **A**) or $c_{\mathbf{B}}$ (criterion for presence of feature **B**), respectively. Thus, with analogous notation as for the marginal d''s, we define the marginal β's as

$$\beta_{\mathbf{A}\mathbf{B}j} = \frac{g_{2j}(c_{\mathbf{A}})}{g_{1j}(c_{\mathbf{A}})}, \qquad j = 1,2;$$

and

$$\beta_{\mathbf{A}i\mathbf{B}} = \frac{g_{i2}(c_{\mathbf{B}})}{g_{i1}(c_{\mathbf{B}})}, \qquad i = 1,2.$$

Signal Detection Microanalyses

In this section we consider the *conditional* d' and β parameters. As for the macroanalyses, the following theoretical formulas correspond to the methods of estimation in earlier experimental papers (Townsend et al., 1984, 1988). Note that these derivations demand that decisional separability holds—that is, that the criteria be parallel to the coordinate axes.

Recall that X is the random variable representing the perceptual effect resulting from the visually degraded presentation of feature **A**, and Y is the random variable analogously defined for feature **B**. We now wish to derive a joint density of X and Y when we condition, for example, on the hit or miss of feature **B**. The hit or miss on feature **B** demands that **B** was actually present in the stimulus; thus, we are concerned with the two densities f_{i2}, i = 1,2. The normalized joint densities for conditioning on the hit of feature **B**, for i = 1 and 2, are written as

$$f_i^*(x, y|\mathbf{B}\text{ hit}) = f_{i2}(x, y) \Big/ \int_{c_\mathbf{B}}^{\infty}\int_{-\infty}^{\infty} f_{i2}(x, y)\, dx\, dy$$

$$= \begin{cases} f_{i2}(x, y) \Big/ \int_{c_\mathbf{B}}^{\infty} g_{i2}(y)\, dy, & \text{for all real } x;\quad y > c_\mathbf{B}\ , \\ 0, & \text{elsewhere}\ , \end{cases}$$

and similarly when we condition on the miss of feature **B**, for i = 1,2,

$$f_i^*(x, y|\mathbf{B}\text{ miss}) = f_{i2}(x, y) \Big/ \int_{-\infty}^{c_\mathbf{B}}\int_{-\infty}^{\infty} f_{i2}(x, y)\, dx\, dy$$

$$= \begin{cases} f_{i2}(x, y) \Big/ \int_{-\infty}^{c_\mathbf{B}} g_{i2}(y)\, dy, & \text{for all real } x;\quad y < c_\mathbf{B}\ , \\ 0, & \text{elsewhere}\ , \end{cases}$$

Here $c_\mathbf{B}$ is the (y-coordinate) value of the decision criterion for feature **B**, X ranges over the real line in both densities, and Y ranges from the criterion $c_\mathbf{B}$ to infinity when conditioning on **B** hit and from negative infinity to $c_\mathbf{B}$ when we condition on **B** miss.

Using these densities, we obtain the corresponding marginal densities for X or Y. The "marginal" conditional density for X conditioned on the hit of feature **B** is obtained by integrating over all possible y values; thus,

$$g_i^*(x|\mathbf{B}\text{ hit}) = \int_{c_\mathbf{B}}^{\infty} f_i^*(x, y|\mathbf{B}\text{ hit})\, dy$$

$$= \int_{c_\mathbf{B}}^{\infty} f_{i2}(x, y)\, dy \Big/ \int_{c_\mathbf{B}}^{\infty} g_{i2}(y)\, dy\ .$$

Similarly, the density for X conditional on the miss of feature **B** is

$$g_i^*(x|\mathbf{B}\text{ miss}) = \int_{-\infty}^{c_\mathbf{B}} f_{i2}(x, y)\, dy \Big/ \int_{-\infty}^{c_\mathbf{B}} g_{i2}(y)\, dy\ .$$

Analogous definitions hold for the marginal densities of the random variable Y conditioned on the hit or miss of feature **B** by integrating the joint densities over all possible x values. Using densities $f_{i1}(x, y)$, i = 1,2 in the integrals gives the joint and marginal densities for conditioning on the false alarm of feature **B** (those that are above $c_\mathbf{B}$) or conditioning on the correct rejections of feature **B** (those below $c_\mathbf{B}$).

By exchanging the roles of **A** and **B**, we also obtain the corresponding definitions for feature **B** conditioned on: (a) the hit or miss of feature **A** by employing joint densities f_{2j}, j = 1,2; or (b) the correct rejection or false alarm of feature **A** with densities f_{1j}, j = 1,2. In these cases, integration is over X, either above or below $c_\mathbf{A}$.

Using these densities, the conditional d''s are defined analogously to the d' parameter of Signal Detectability Theory. For example, the d' value for feature **A** conditional on the hit of feature **B** is given by

$$d'(\mathbf{A}|\mathbf{B}\text{ hit}) = \frac{E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_2\mathbf{B}_2] - E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_1\mathbf{B}_2]}{[\frac{1}{2}(Var\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_2\mathbf{B}_2] + Var\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_1\mathbf{B}_2])]^{1/2}}\ ,$$

where $E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]$ and $Var\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]$ are the expected value and variance of X, respectively, given that feature **B** was a hit and stimulus $\mathbf{A}_i\mathbf{B}_2$ was presented. The expected values, for i = 1 and 2, are defined by

$$E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = \int_{-\infty}^{\infty} x g_i^*(x|\mathbf{B}\text{ hit})\, dx$$

$$= \int_{c_\mathbf{B}}^{\infty} g_{i2}(y) E\,[X|Y = y, \mathbf{A}_i\mathbf{B}_2]\, dy \Big/ \int_{c_\mathbf{B}}^{\infty} g_{i2}(y)\, dy\ .$$

But for two random variables, X and Y, having a bivariate normal distribution, the conditional mean of X at a given y value is

$$E\,[X|Y = y] = \mu_X + (y - \mu_Y)\,\rho\,\frac{\sigma_X}{\sigma_Y}\ ,$$

where ρ is the correlation coefficient of the bivariate normal density, μ_X and σ_X^2 are the mean and variance of the marginal density of X, respectively, and similarly for μ_Y and σ_Y^2. Substituting this into the preceding integral and simplifying gives

$$E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = \mu_{Xi2} - \rho_{i2}\,\frac{\sigma_{Xi2}}{\sigma_{Yi2}}\,(\mu_{Yi2} - E\,[Y|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2])\ ,$$

where

$$E\,[Y|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = \int_{c_\mathbf{B}}^{\infty} y g_i^*(y|\mathbf{B}\text{ hit})\,dy$$

$$= \int_{c_\mathbf{B}}^{\infty} y g_{i2}(y)\,dy \Big/ \int_{c_\mathbf{B}}^{\infty} g_{i2}(y)\,dy\,.$$

For the denominator of $d'(\mathbf{A}|\mathbf{B}\text{ hit})$,

$$Var\,[X|B\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = E\,[X^2|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] - \{E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]\}^2\,,$$

with

$$E\,[X^2|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = \int_{-\infty}^{\infty} x^2 g_i^*(x|\mathbf{B}\text{ hit})\,dx$$

$$= \int_{c_\mathbf{B}}^{\infty} g_{i2}(y) E\,[X^2|Y = y, \mathbf{A}_i\mathbf{B}_2]\,dy \Big/ \int_{c_\mathbf{B}}^{\infty} g_{i2}(y)\,dy$$

and $E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]$ as before. But $E\,[X^2|Y = y, \mathbf{A}_i\mathbf{B}_2] = Var\,[X|Y = y, \mathbf{A}_i\mathbf{B}_2] + \{E\,[X|Y = y, \mathbf{A}_i\mathbf{B}_2]\}^2$, and $Var\,[X|Y = y] = \sigma_X^2\,(1 - \rho^2)$, so

$$Var\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2] = \sigma_{Xi2}^2\,[1 - \rho_{i2}^2] + \rho_{i2}^2 \frac{\sigma_{Xi2}^2}{\sigma_{Yi2}^2}\,Var\,[Y|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]\,.$$

Similar expressions are defined for all conditional d''s. For example,

$$d'(\mathbf{A}|\mathbf{B}\text{ miss}) = \frac{E\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_2\mathbf{B}_2] - E\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_1\mathbf{B}_2]}{[\frac{1}{2}(Var\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_2\mathbf{B}_2] + Var\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_1\mathbf{B}_2])]^{1/2}}\,,$$

where $E\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_i\mathbf{B}_2]$ and $Var\,[X|\mathbf{B}\text{ miss}, \mathbf{A}_i\mathbf{B}_2]$ are defined identically to $E\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]$ and $Var\,[X|\mathbf{B}\text{ hit}, \mathbf{A}_i\mathbf{B}_2]$ (using "**B** miss" in place of "**B** hit"). Analogous definitions are obtained for d''s conditional on correct rejections and false alarms by using the appropriate densities.

The definitions for the conditional β's are easier to derive. Using the "marginal" conditional densities, we simply write the conditional β for feature **A** given the hit or miss of feature **B** as the ratio

$$\beta(\mathbf{A}|\mathbf{B}\text{ hit}) = \frac{g_2^\dagger(c_\mathbf{A}|\mathbf{B}\text{ hit})}{g_1^\dagger(c_\mathbf{A}|\mathbf{B}\text{ hit})}$$

and

$$\beta(\mathbf{A}|\mathbf{B}\text{ miss} = \frac{g_2^\dagger(c_\mathbf{A}|\mathbf{B}\text{ miss})}{g_1^\dagger(c_\mathbf{A}|\mathbf{B}\text{ miss})}\,,$$

where $c_\mathbf{A}$ is the x coordinate of the criterion for reporting feature **A**.

MULTIDIMENSIONAL MODELS OF PERCEPTION AND COGNITION

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