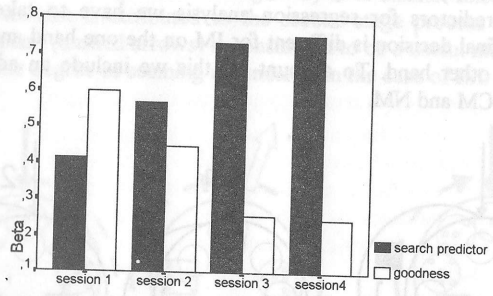

Fig.4. Explanation in the text.

References

Buffart, H. & Geissler, H.-G. (1984). Task-dependent representation of categories and memory-guided inference during classification. In Degreeef, E. & Buggenhaut, J. (Eds.), New Trends in Mathematical Psychology, 33-58. Amsterdam: North-Holland.

Chekosky, S.F. & Witlock, D. (1973). The effects of pattern goodness on recognition time in a memory search task. Journal of Experimental Psychology, 100, 341 - 348.

Garner, W.R. & Clement, D.E. (1963). Goodness of pattern and pattern uncertainty. Journal of Verbal Learning and Verbal Behavior, 2, 446-452.

Geissler, H.-G. (1980). Dynamic Frames of Reference in Judgement and Recognition. In F. Klix and B. Krause (Eds.), Humboldt University 1960 - 1980, 53-83. Berlin: Deutscher Verlag der Wissenschaften.

Geissler, H.-G. (1985). Sources of seeming redundancy in temporally quantized information processing. In G. d'Ydewalle (Ed.), Sel./rev. papers of the 23rd ICP, Vol. 3, 119- 228. Amsterdam: North-Holland.

Geissler, H.-G. (1987). Guided Inference: Components of Task-Dependence in Human Information Processing. In van der Meer, E. & Hoffmann, J. (Eds.), Knowledge Aided Information Processing, 221-240. Amsterdam: North-Holland.

Geissler, H.-G. (1991). Constraints of mental self-organization: The indirect validation approach toward perception. Estratto da Comunicazioni Scientifiche di Psicologia Generale 5, 47-69.

Geissler, H.-G. (1995). Beziehungserkennung und interne Suche - zur Architektur und einem besonderen Fall struktureller Erkennung. In Pawlik, K. (Ed.): Report on the 39th Congress of the Deutsche Gesellschaft für Psychologie (in German), 570 - 575. Göttingen: Hogrefe.

Geissler, H.-G., Puffe, M. & Stern, W. (1982). Item-recognition and no end: Representation Format and Processing Strategies. In Geissler, H.-G. and Petzold, P. (Eds.), Psychophysical Judgement and the Process of Perception, 270-281. Berlin: Deutscher Verlag der Wissenschaften.

Geissler, H.-G. & Puffe, M. (1983). The Inferential Basis of Classification: From Perceptual to Memory Codes. In Geissler, H.-G. et al (Eds.), Modern Issues in Perception, 87-124. Amsterdam: North-Holland.

Lachmann, Th. (1996). Informationsverarbeitung beim visuellen Vergleich. Beiträge zur wissenschaftlichen Diskussion 15. München: Förderwerk Hanns-Seidel-Stiftung.

Schmidt, K.-D. & Ackermann, B.(1990). The Structure of Internal Representations and Reaction-Time Related Matching Task Phenomena. In Geissler, H.-G. (Ed.), Psychophysical Explorations of Mental Structures, 278-289. Toronto a.o.: Hogrefe & Huber Publishers.

Zandt, T. van & Townsend, J.T. (1993). Self-terminating versus exhaustive processes in rapid visual and memory search: An evaluative review. Perception & Psychophysics, 53 (5), 563-580.

PATTERNS AND FEATURES AS MANIFOLDS: FACES IN IDENTIFICATION AND CATEGORIZATION

James T. Townsend & Michael J. Wenger
Indiana University

Abstract

The present chapter considers some of the possibilities associated with the use of infinite dimensional spaces in perceptual and cognitive theorizing. Of particular import and application are questions in face perception and cognition, a domain in which the use of infinite dimensional spaces would appear particularly advantageous, given the complexities (geometric, perceptual, and cognitive) associated with faces (more generally, heads). We outline suggestions for initial applications, both mathematical and experimental.

Despite the numerous examples in perception and cognition in which the objects of information processing could not be simple finite dimensional vector spaces, much of our theory and technology of measurement, scaling and psychophysics is devoted almost entirely to these. (Interestingly, as an aside, outside of the assumption of orthogonality and imposition of a metric, the vital assumptions of vector spaces are hardly ever employed; but that is beyond our present scope of discussion.) Townsend and Thomas (1993) provide references to exceptions to this general rule. In no way do we disparage the great importance of these spaces. However, we do believe it is high time to enlarge our perspective to include some of the rich possibilities that modern mathematics offers.

As an example that is intended to guide ourselves as well as to illustrate what we mean, we call on face processing. We propose as a hypothesis that identification of faces and categorization may follow somewhat different laws. First, though, we need to sketch in, in a mostly qualitative fashion, the type of mathematics that are applicable. Clearly, there is insufficient space for any degree of detail.

We start with the non-controversial concept of mappings (loosely, functions) between spaces to represent various levels of perceptual and cognitive processing. What these mappings can preserve, distort, etc., depends not only on the domain and range spaces, but the properties of the map, of course. Hence immediately we see that even the entire image (or more abstract representation as the case may be) is a pale simulacrum of the original object in the real world . That is, even at a relatively macroscopic level, much information must be lost in energy transduction of the narrow band and type of energy impacting the sensorium, not to mention the coarser and finer grained depictions (e.g., molecular and atomic description) that are unavailable to our perceptual mechanisms.

We then, again for most folks in an indisputable way, view a feature of an object as a map of the original object to some space or subspace. In this sense, the global perceptual object is simply the maximal feature representable by the perceiver.

Now we can get a bit more specific and take up the case of face perception. We must naturally ignore the great number of interesting theories and experiments in this burgeoning sphere (e.g., Bruce, 1988; Bruce & Humphreys, 1994; Tanaka & Farah, 1991) in order to focus on our objective. Suppose that faces, or more generally heads, as perceptual objects are manifolds (to avoid pedantry, we will drop constant reference to the original objects vs. their perceptual counterparts except where necessary). What are manifolds? Basically, they are spaces, which locally (e.g., including a neighborhood around an arbitrary point) are Euclidean, that is, can be mapped in a 1-1, bicontinuous and into a region of a finite dimensional Euclidean vector space. With a bit more structure, the differential and integral calculus can be brought into the picture and we then speak of differential manifolds and differential geometry. (For those interested, the difference between differential geometry and differential topology is that the latter requires, and utilizes geometric topics such as angle between vectors, whereas the latter depends only on more general spatial aspects ; but there is much cross-talk across the areas.)

That a head, as original object and provisionally as perceptual representation can be pictured as a manifold follows immediately from approximating it by a continuous distortion of a sphere (ignoring, if their are any algebraic topologists in the audience, the "tear" distortions created by holes; not a crucial distortion of the facts for present purposes).

Now, a fair way into any decent text on differential geometry one encounters the concept of the first fundamental form (in a book on tensors, usually earlier than that). A "form" here is a kind of map that carries vectors into the real numbers, in a sense a kind of measure (e.g., feature?) on the manifold. There are several ways in which to conceive of this form. For our purposes, perhaps the

most direct approach is to think of it as a means of computing lengths of paths on the manifold (which of course may be very non-flat). Recall that even in ordinary calculus, we treat velocity as a vector and calculate path length as an integral of that velocity over a path. In general, we first have to map a local region of our manifold into a Euclidean space, if we want to take an approach that includes specification of coordinates (it is possible to carry out derivations without these in most cases). However, certainly a head can be embedded (in a technical as well as colloquial sense), in a three-dimensional space, so we can just use those coordinates.

In differential manifolds, we can also view velocity as a vector, in this case as a three-dimensional vector, but how we convert this to a metric that takes into account the special properties of our particular manifold is where the first fundamental form comes in. Basically, we have this form written as $\mathcal{F}_1 = g_{ij}\frac{\partial x_i}{\partial t}\frac{\partial x_j}{\partial t}$, where it is plain to see that the velocities play an important role. The neat thing, though, is the g_{ij}, which tells how the space is changing as we move from point to point. (Actually, according to convention, the ∂t s are usually omitted, since we can integrate across the space without explicit account of them, as in line integration in advance calculus!). This first fundamental form not only delivers path lengths, it virtually completely determines the so-called intrinsic characteristics of a manifold. For instance, if we lived on a planet which was forever fog-enshrouded and assuming we had no space exploration at our finger tips, all that we could ever determine about the surface aspects would be specified by this form. Nevertheless, this very powerful feature" is not sufficient to explain face identification. For instance, a sheet of paper laying flat on a table and the same sheet of paper rolled into a cylinder possess the same first fundamental form; yet, we can tell the difference!

Neglecting for the time being the problem of just noticeable differences as well as obvious constancies such as translation and dilation invariances, we can ask what would be required for complete geometric congruence (in the same precise sense that we learned in high-school geometry). The answer is the second fundamental form. We haven't much space to explicate this form, but basically, it is an extrinsic function, therefore one that is not completely determined by the first fundamental form, or map (incidentally also one that maps pairs of vectors as does the first form) that measures how any vector on the manifold is changing as we move along a path. In this sense, it is measuring something like our concept of acceleration which is itself fundamentally related to curvature of the manifold. Its expression is $\mathcal{F}_2 = f_{ij}\frac{\partial x_i}{\partial t}\frac{\partial x_j}{\partial t}$, which looks rather like the first fundamental form. However, f_{ij} is gauging how a vector that is orthogonal (i.e., normal") to the surface" of the manifold (the latter being an extrinsic, not

an intrinsic, notion) is varying as one move in different directions away from the current position.

Anyhow, agreement of these two forms together is sufficient to ensure congruence. Now of course, these are only two of a very large number of structurally equivalent) encoding possibilities. Nevertheless, they do permit the description of shapes, or much more abstract spatial objects, in a way not imprisoned by finite dimensional vectors (although naturally the latter play a dynamic role in the overall proceedings of differential geometry), and in a way that completely specifies an object. We also know, of course, that many perceptual studies have been accomplished that demonstrate deficiencies of observers and therefore an ideal models of this sort, in their ability to veridically perceive a number of aspects of three-dimensional space (e.g., Braunstein, Liter, & Tittle, 1993; Todd, 1989). Nevertheless, we believe to make progress in object and face cognition, that we must work with the tools at hand. As time goes by, the experimental confines can be built into emerging models, possibly employing new or at least different geometric and topological concepts.

We plan to begin experimentation implementing and testing models based on differential geometry and other relatively novel mathematical objects such as function spaces and relevant metrics on those spaces. One goal will be to start determining just-noticeable-differences and other measures of resolution, using such notions, in face perception.

References

Braunstein, M. L., Liter, J. C., & Tittle, J. S. (1993). Recovering three-dimensional shape from perspective translations and orthographic rotations. *Journal of Experimental Psychology: Human Perception and Performance, 19*, 598-614.

Bruce, V. (1988). *Recognising faces.* Hillsdale, NJ: Erlbaum.

Bruce, V., & Humphreys, G. (1994). *Object and face recognition.* Hillsdale, NJ: Erlbaum.

Tanaka, J. W., & Farah, M. J. (1991). Second-order relational properties and the inversion effect: Testing a theory of face perception. *Perception and Psychophysics, 20*, 367-372.

Todd, J. T., & Reichel, F. D. (1989). Ordinal structure in the visual perception and cognition of smoothly curved surfaces. *Psychological Review, 96*, 643-657.

Townsend, J. T., & Thomas, R. D. (1993). On the need for a general quantitative theory of pattern similarity. In S. C. Masin (Ed.), *Foundations in perceptual theory: Advances in psychology, Vol. 99* (pp. 297-368). Amsterdam: North-Holland.