# CORRELATED DYNAMIC LINEAR ACCUMULATORS WITH GAUSSIAN NOISE: INITIAL PREDICTIONS FOR TARGET REDUNDANCY

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Abstract

We present an initial investigation of the impact of channel interactions in dynamic, stochastic models for target redundancy. Predictions are detailed for early and late inhibitory and facilitatory interactions with respect to distributions of finishing times, violations of the Miller and Grice inequalities, and a fine-grained measure of process capacity.

Consider a task where either no target or a single target is presented, where a negative response is given to the former and an affirmative response to the latter. Compare that with a situation where two or more targets can be presented on affirmative trials and the observer responds if *any* target is detected. Perceptual and sensory scientists have long been interested in the degree to which performance is improved in the multi-target (or redundant targets) case. The answer to this question bears on issues such as architecture (e.g., serial vs parallel processing), stopping rule (e.g., self-terminating vs exhaustive processing), capacity, and independence. The evidence is overwhelming that observers are faster when two or more targets are presented than when a single target is presented. This suggests either parallel self-termination or some type of convergence of information of several channels. If the improvement is great enough, Miller (e.g., 1982) suggested that ordinary parallel processing will not suffice to explain the results and something he referred to as "coactivation" must occur.

Until Miller's papers, the basis for rejecting ordinary parallel processing was the comparison with predictions from a parallel model assuming (a) context independence (or invariance), (b) stochastic independence and (c) self-termination. Context invariance, overlooked as an important facet until the mid-1980s (e.g. Ashby & Townsend, 1986; Luce, 1986), assumes that the marginal response time distributions for the various channels) are unchanged from single to multi-target trials and implies unlimited capacity. Unlimited capacity is defined as the invariance of the marginal processing distribution associated with a single item as more and more items are added to the processing load. Stochastic independence assumes that the probability that neither channel has finished its job by some time t, is just the product of the separate probabilities. Finally self-termination allows multi-target trials to be faster simply due to the statistical law that the minimum of a set of independent random variables will be stochastically less than when fewer are observed. This latter result is referred to as the "horse race" or "race" prediction and is also known as "probability summation." Probability summation under the preceding three assumptions takes the form

$$\begin{aligned} P_{12}(T_{12} \leq t) &= P_{12}(T_1 \leq t) + P_{12}(T_2 \leq t) - P_{12}(T_1 \leq t) \cdot P_{12}(T_2 \leq t) \\ &= P_1(T_1 \leq t) + P_2(T_2 \leq t) - P_1(T_1 \leq t) \cdot P_2(T_2 \leq t) \end{aligned} \quad (1)$$

where the (1,2) subscript denotes redundant target trials and the $i (i = 1, 2)$ subscript implies a single target trial. Miller (e.g. 1982) noticed that equation [1] can be made into a stronger test. First, observe that even if independence is wrong, it still must be the case that, assuming context invariance and self-termination,

$$P_{12}(T \leq t) = P_1(T_1 \leq t) + P_2(T_2 \leq t) - P_{12}(T_1 \leq t \cap T_2 \leq t) \quad (2)$$

Now, since $P_{12}(T_1 \leq t \cap T_2 \leq t)$ always diminishes the overall quantity, it must be the case that

$$P_{12}(T \leq t) \leq P_1(T_1 \leq t) + P_2(T_2 \leq t) \quad (3)$$

This inequality is known generally as Boole's inequality and in psychology as the "race model inequality" or "Miller's inequality." Miller pointed out that if the inequality is violated, some kind of rather extraordinary processing must be occurring, which as noted, he referred to as coactivation. In a theory developed by Townsend and Nozawa (1995), violation of Miller's inequality is associated with super capacity. Super capacity is defined as performance over and above that predicted by unlimited capacity, independent parallel processing. Violation of Miller's inequality implies super capacity and super capacity over a substantial interval, implies a violation of Miller's inequality (Townsend & Nozawa, 1997). There is another inequality which tests the lower limits of capacity, rather than the upper limits. Grice and colleagues (Grice, Canham & Boroughs, 1984) were apparently the first to suggest its use (actually in a study of distractor effects), so we have referred to it as Grice's inequality (Townsend & Nozawa, 1995), and its expression is

$$P_{12}(T \leq t) \geq \min[P_1(T_1 \leq t), P_2(T_2 \leq t)] \quad (4)$$

Intuitively, it says that ordinary parallel processing should not be slower than the fastest single channel performance. As one might expect, it turns out to be indicative of highly limited capacity within our formal theory.

Colonius (1990) later linked the Miller and Grice inequalities with results originally put forth by the well-known mathematician René Frechet. The basic idea for our purposes is as follows: Assume that the marginal distributions on each item are constrained to remain unchanged from single to double target trials. Then the rather extraordinary coincidence occurs that the upper and lower limits on speed, interpreted probabilistically, the so-called Frechet bounds, are identical to the Miller and Grice inequalities! Now, if independence were assumed, then of course, nothing exceeding probability summation could occur under these conditions, and the bounds could not be approached. However, if stochastic dependence is allowed then the best possible performance, which can happen under a perfect negative dependence between channel processing times is given by the Miller bound. Conversely, when a perfect positive dependence is present, the worst case scenario occurs and the Grice bound is reached.

### When real-life channels interact

Although previous work has greatly clarified the situation regarding essential processing issues in redundant targets paradigms, it remains to be investigated what happens with real systems. That is, the attainment of the Miller and Grice bounds occurs under the constraint that the marginal distributions remain invariant as the number of engaged channels increases. Does this occur in actual systems? And, what do we mean by "actual systems?"

Concepts about processing can be instantiated in models that are framed in terms of some measure that specifies the state of processing that has occurred up until any arbitrary time. It appeared likely to us that when models are constrained by actual interactions involving their states of activation, the condition of context invariance might not be met. That is the focus of this incipient study. Moreover, we were intrigued by the fact that while positive correlations tend to decrease speed in a redundant targets experiment, when context invariance is in force, we suspected that this could be more than made up for by a potential increase (violation of context invariance) in the marginal distribution functions. Likewise, although negative dependence tends to increase redundant target performance (because it subtracts less than the multiplicative term in the unlimited capacity, independent parallel model), it could decrease the marginal probabilities so much that an actual diminution in performance (i.e., slower response times) might be found. Indeed, previous models (e.g. Mordkoff & Yantis, 1991) as well as general theoretical reasoning (e.g. Colonius & Townsend, 1997) support this view. Nevertheless, there has been no direct study of this issue until the present investigation.

A prototypical parallel model is based on the pair of linear differential equations

$$\dot{x}_1 = ax + by; y_1 = cx + dy \quad (5)$$

$$\dot{x}_2 = bx + ay; y_2 = cy + dx \quad (6)$$



The differential equations represent rate of activation in the two channels, respectively. Note that the $x$s represent the state activations and the $y$s represent the outputs. The $a$ parameter indicates the degree of feedback within the respective channels whereas $b$ represents the degree of "cross-talk" across channels (we refer to this as *early* crosstalk); $c$ and $d$ are the parameters representing output weighting that is within, or across, channels respectively (*late* crosstalk). We have chosen to start with the case of symmetry in that the within-channel and cross-channel feedback and output weightings are identical, and this can obviously be easily generalized. In each case, if the cross-talk parameter is positive, this implies that the channels increase each other's activation, that is, facilitation is occurring. If the cross-talk parameters are negative than inhibition is in force.

In order to permit a stochastic depiction of activation, we induce noise in the form of added Gaussian noise, before integration of activation occurs. It is important to note though, that this Gaussian noise is a true stochastic process, not just an addition of a Gaussian random variable. Finally, we impose activation thresholds on each channel.

It is obviously the early or late cross-talk that can lead to inter-channel correlations. In order to focus on the early vs late channel interactions, we alternatively let $b = 0$, that is the early cross-talk is eliminated but not the late, or $d = 0$, that is the late channel interaction is eliminated, but not the early. The system that includes only late interaction will be referred to as System I and that with only early interaction will be referred to as System II.

Note that because only one cross-talk parameter, $b$ or $d$, is employed in early or late processing, only facilitation or inhibition is occurring at each level. Furthermore, because we isolate the types of early vs. late interaction, we can have no trade-off of facilitation vs inhibition. Another natural constraint that we impose is that the systems are always stable. This simply means that the system can never run off toward infinity. Furthermore, we assumed that the internal weight in each channel's output, $c > 0$. In the event of facilitatory early interaction, $b > 0$, the effect of $a < 0$ must be bigger than the combined effect of $b > 0$. The parameter values were selected to obey these conditions.

Predictions of our systems were obtained by simulation, and those predictions were compared on $F(t)$ (the cdf of finishing times), Miller's and Grice's inequalities, and the finer-grained capacity coefficient $C(t)$. $C(t) > 1$ indicates super capacity, $C(t) < 1$ indicates limited capacity, and $C(t) = 1$ indicates unlimited capacity. In the following, we will refer to the prediction garnered by taking the single target performance and assuming independent, unlimited capacity, self-terminating (i.e., stopping when the first completion occurs) processing as the "race prediction".

### Results

Consider first System I when the crosstalk is inhibitory. In this case, the inhibitory effects are seen to actually slow down the redundant trial performance to such an extent that performance is worse than that predicted by the single $F(t)$s. Miller's inequality is satisfied, Grice's inequality is violated, and $C(t) < .5$, indicating incredibly low capacity. Now consider System I when the crosstalk is facilitatory. Contrary to the preceding case, here the redundant target performance greatly exceeds that predicted by the race model. Now the Grice bound is never crossed, Miller's bound is violated, and $C(t)$ indicates extreme super capacity.

For System II when the crosstalk is inhibitory, performance is worse than the race prediction, but does not fall below the single target prediction. Miller's inequality is satisfied, but performance sits right atop the Grice bound. The interpretation of this result is that capacity is fixed, an interpretation that is reinforced by $C(t)$, which hovers closely around $C(t) = .5$. For this system when crosstalk is facilitatory, performance is faster than that found with the race model, Grice's inequality is never violated, Miller's inequality is violated, and $C(t)$ is much greater than 1, indicating super capacity.

### Discussion and conclusion

The upshot of our simulations with parallel linear systems with decision threshold and intruding Gaussian noise is as follows: Inhibitory influences, whether created by cross-feedback or late effects on output, can and do cause sufficient harm to processing speed that the mild helpful effects on the dependence term (i.e., $P_{12}(T_1 \leq t \cap T_2 \leq t)$) are more than overwhelmed by the imposed massive inhibition. The latter effect can be viewed as occurring in the marginal distribution functions—that is, context invariance is not obeyed and in fact outweighs the dependence consequence itself.

The same can be said of facilitatory cross-talk in reverse: The slight hurt to speed as captured by the fact that $P_{12}(T_1 \leq t \cap T_2 \leq t)$ is less than $P_1(T_1 \leq t) \cdot P_2(T_2 \leq t)$ in the cumulative probability of stopping by a certain time in the redundant targets formula, is dominated by the positive effect on the marginal distribution functions. Thus, again context invariance fails in a way that makes the performance move in the opposite direction to the tendency caused by the dependency itself.

It is obvious that different parameter values could change the degree of effect of positive or negative interactions. The present investigation simply proves that such interactions can affect performance in a powerful fashion, violating context invariance in the process. What has not been shown presently, is what the lower and upper bounds on performance are, if any, created by positive and negative interactions. Nevertheless, our suspicions are that the assumption of stability creates a kind of asymmetry in the following sense. Because the system can stay stable even with tremendous negative cross-talk, it seems likely that performance can be made very slow indeed. Only by restraining the interactions to realistic levels (e.g., b and d not being larger in the negative direction than is a) will any bounds on change from single to double target conditions obtain. On the other hand, since positive lateral interactions "plus" the positive effect of c, must always be outweighed by the parameter a to ensure stability, there may be a more profound limit on how much positive interactions can help.



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